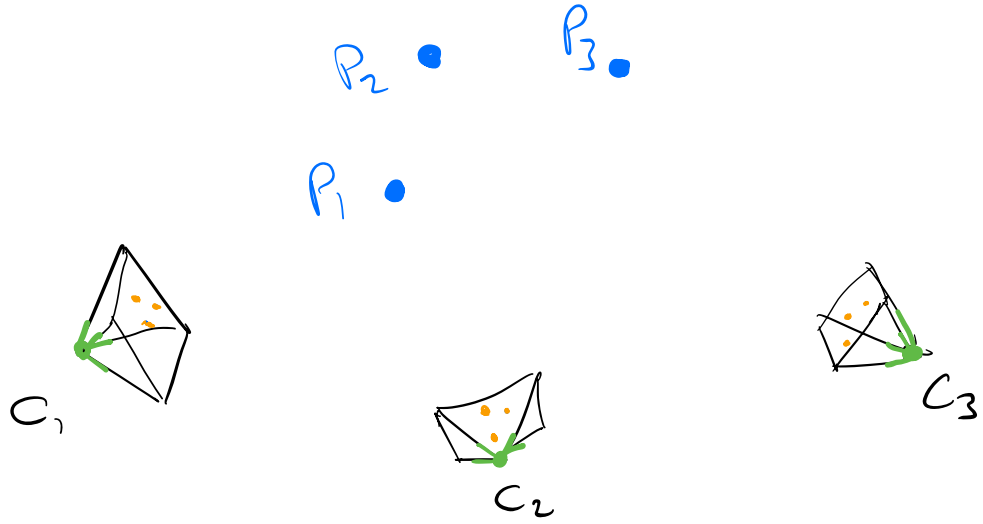
3D Problem Taxonomy



	"Structure" (3D scene points)	"Motion" (camera pose)	Measurements Needed
Pose Estimation	known	?	2D-3D correspondences
Triangulation	?	known	2D-2D correspondences
Structure from Motion	?	?	2D-2D correspondences

$$\begin{bmatrix} \hat{x}_i \\ \hat{y}_i \\ 1 \end{bmatrix} = \begin{bmatrix} x_p/w_p \\ y_p/w_p \\ 1 \end{bmatrix} \sim \begin{matrix} x_p \\ y_p \\ w_p \end{matrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ W_i \end{bmatrix}$$

2D obs ↓ 2D reprojection ↓

$$r_i^x = x_i - \hat{x}_i$$

$$r_i^y = y_i - \hat{y}_i$$

(Skip?)

Epipolar Geometry

Assume:

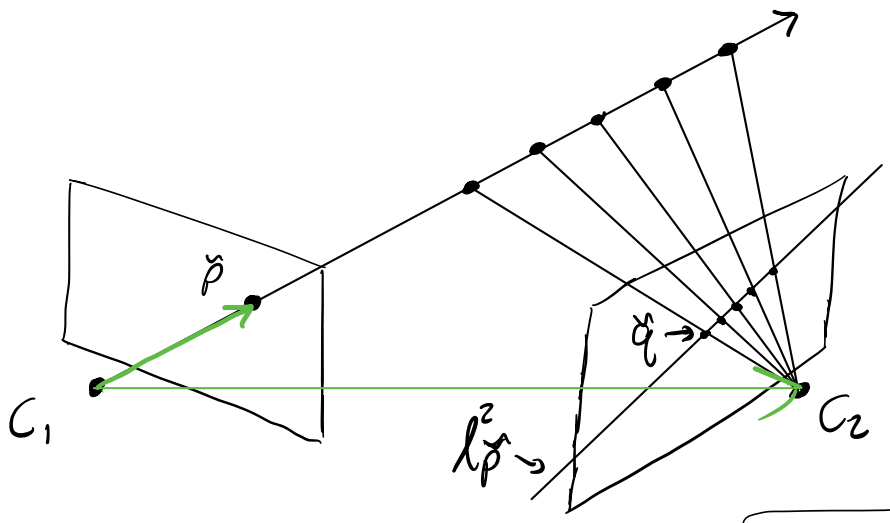
$K_1 = I_{3\times3}$

$K_2 = I_{3\times3}$

$R_1 = I_{3\times3}$

$t_1 = \vec{0}$

R_2, t_2 known



$$R_2(t_2 \times \hat{p})$$

$$l^2_{\hat{p}} = R_2 [t]_\times \hat{p}$$

$$\hat{q} \cdot l^2_{\hat{p}} = 0$$

$$\rightarrow \hat{q}^T [R_2 [t_2]_\times] \hat{p} =$$

Epipolar Constraint: $\hat{q}^T E \hat{p} = 0$

↖ The Essential Matrix

Aside: Cross Product ↓ Matrix Multiply

$t \times p$

↳ $[t]_\times \vec{p}$

↑

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{3\times3}$$

camera ↓ pixel ↓

Let $\hat{p} = K_1^{-1} p$

$\hat{q} = K_2^{-1} q$

$\hat{q}^T$

$q^T [K_2^{-T} \; R_2 [t_2]_\times \; K_1^{-1}] p$

$$q^T F p = 0$$

↖ The Fundamental Matrix

DLT - direct linear transform

Reprojection Error residuals:

$$r_x^i \left(p_{20} X_i + p_{21} Y_i + p_{22} Z_i + p_{23} W_i\right) x_i - p_{00} X_i + p_{01} Y_i + p_{02} Z_i + p_{03} W_i$$

$$r_y^i \left(p_{20} X_i + p_{21} Y_i + p_{22} Z_i + p_{23} W_i\right) y_i - p_{10} X_i + p_{11} Y_i + p_{12} Z_i + p_{13} W_i$$