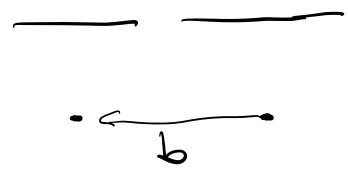
Plane Sweep Stereo



Setting:

Cameras are <u>not</u> rectified

Cameras <u>are</u> calibrated ($K_l, [R;t]_l, [R;t]_r$, K_r are known)

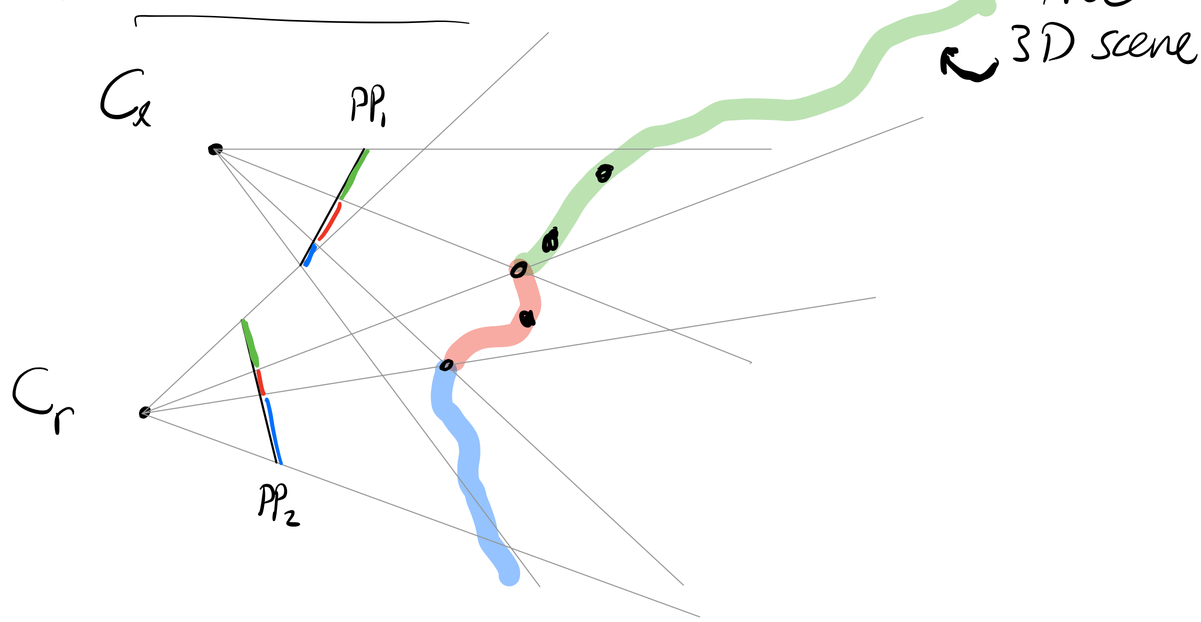
Standard Stereo:

for each pixel
  for each disparity
    compute match cost

Plane Sweep Stereo:

for each depth
  for each pixel
    compute match cost

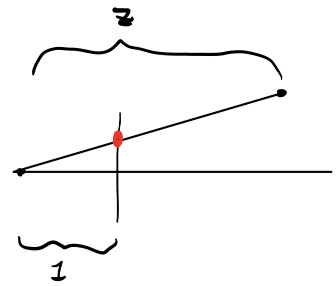
Intuition:
(1) "unproject" a pixel to a hypothesized depth d
(2) "reproject" that 3D point back into the other camera
(3) compute match score



Details:

Given: $K_L^{3\times3}$ $K_R^{3\times3}$ $[R|t]_L^{3\times4}$ $[R|t]_R^{3\times4}$

(1) "unproject":



- convert pixels to camera coords
- move to depth d
- put in world coords

$$\begin{bmatrix} x_w\\ y_w\\ z_w\\ 1\end{bmatrix} \simeq \begin{pmatrix} R & t\\ 0\;0\;0 & 1\end{pmatrix}^{-1} \overset{\text{(augment)}}{\begin{bmatrix} dx_c\\ dy_c\\ d\\ 1\end{bmatrix}} \;\; \underbrace{d\, \underbrace{K_L^{-1} \begin{bmatrix}x_{img}\\y_{img}\\1\end{bmatrix}_L}_{\begin{bmatrix}x_c\\y_c\\1\end{bmatrix}}}_{\begin{bmatrix}dx_c\\dy_c\\d\end{bmatrix}}$$

$$\begin{bmatrix}R^TR & R^Tt - R^Tt\\ 0\;\;0\;0 & 1\end{bmatrix} = \begin{bmatrix}R^T & -R^Tt\\ 0\;\;0\;0 & 1\end{bmatrix}\begin{bmatrix}R & t\\ 0\;0\;0 & 1\end{bmatrix}$$

(2) "Reproject"

Given: $\overset{3\times3}{K_L \;\; K_R}$ $\underline{\overset{3\times4}{[R_L|t_L]}}$ $\overset{3\times4}{[R_R|t_R]}$

$$\begin{pmatrix} R & | & t\\ 0\,0\,0 & & 1\end{pmatrix} = \begin{bmatrix}\vec u & \vec v & \vec w & \vec c\\ 0 & 0 & 0 & 1\end{bmatrix}^{-1}$$

- world to cam
- cam to pixel

$$\begin{bmatrix}x_{img}\\y_{img}\\1\end{bmatrix}_R \sim \begin{bmatrix}x_p\\y_p\\w_p\end{bmatrix} = K_R \underbrace{[R_R|t_R]\begin{bmatrix}x_w\\y_w\\z_w\\1\end{bmatrix}}_{x_c,\; y_c,\; z_c}$$

(3) compute match score

$$\begin{bmatrix}x_{img}\\y_{img}\\1\end{bmatrix}_R = K_R\,[R|t]_R\; d\; [R|t]_L^{-1}\,K_L^{-1}\begin{bmatrix}x_{img}\\y_{img}\\1\end{bmatrix}_L$$

Insight: "unproject-reproject" is a homography!

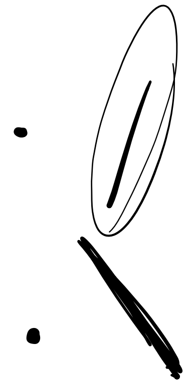
Strategy: (1) unproject

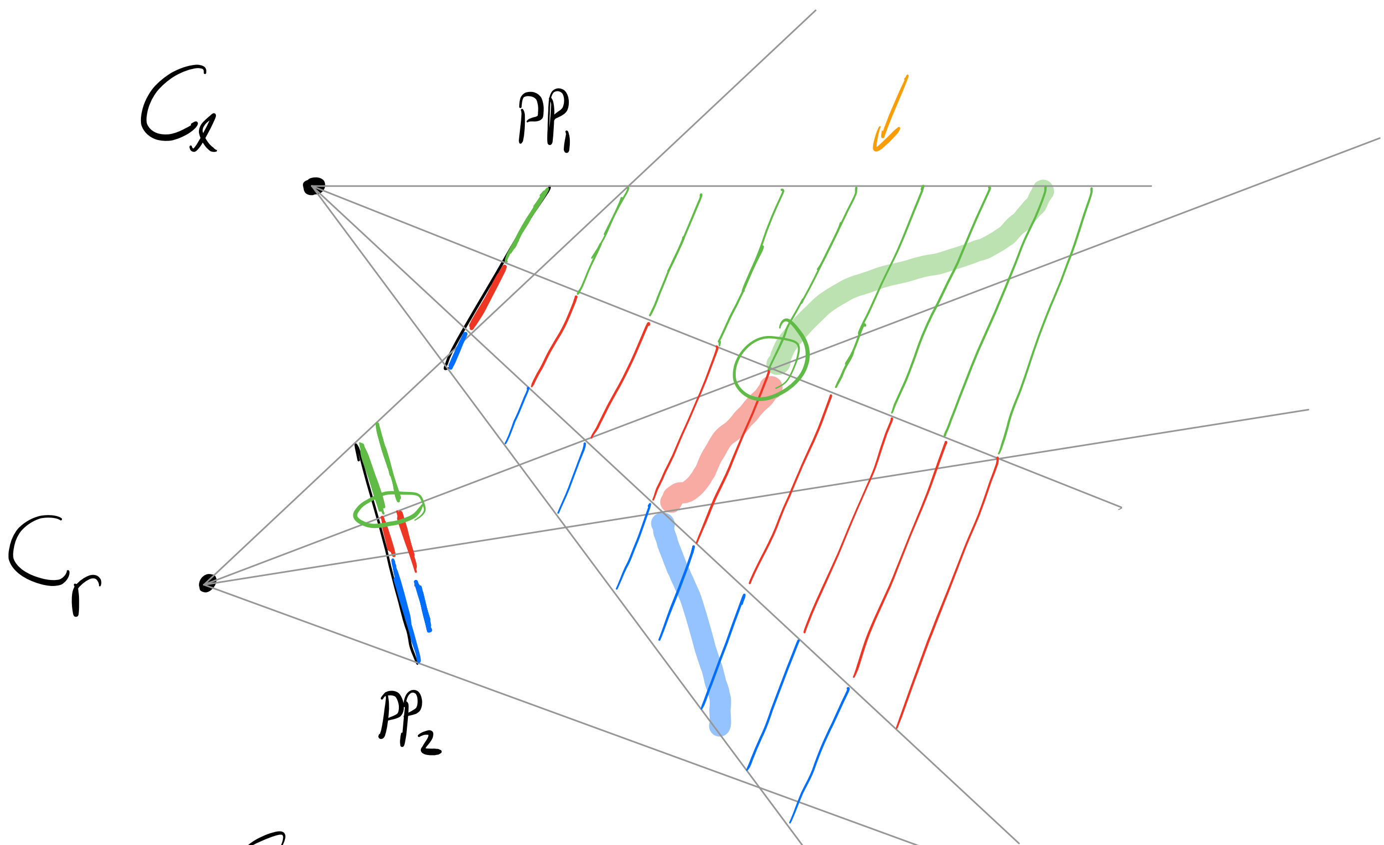
(2) reproject

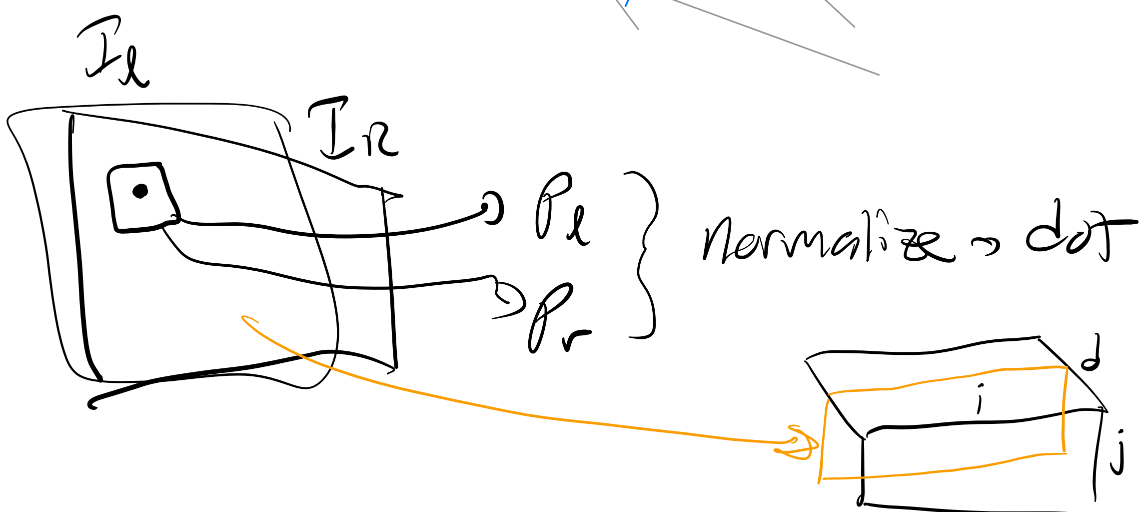
(3) fit H

(4) Warp

(5) Compute NCC







$$[R ; t] = \begin{pmatrix} R & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0\,0\,0 & 1 \end{pmatrix} \begin{pmatrix} I_{3\times 3} & t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{pmatrix} R & R \cdot t \\ 0\,0\,0 & 1 \end{pmatrix}$$

(translate $\underline{\text{then}}$ rotate)

$$\operatorname{inv} \begin{pmatrix} R & R \cdot t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{bmatrix} R^T & -t \\ 0\,0\,0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{bmatrix} R & t \\ 0\,0\,0 & 1 \end{bmatrix}^{-1}$$

$$= \left(\begin{bmatrix} I & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} R^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -t \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R^T & R^T t \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{R} & \hat{t} \\ 0 & 1 \end{bmatrix}^{-1} \overset{\checkmark}{=} \begin{bmatrix} \hat{R}^T & -\hat{R}^T t \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{bmatrix} \hat{R}\hat{R}^T & -\hat{R}\hat{R}^T t + t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$