Lecture 9 - Projective Alignment

Robust Model Fitting

$$\min_T \sum_i \left\| T_{p_i} - p_i' \right\|^2$$

$$\begin{bmatrix} \frac{ax+by+c}{gx+hy+k} \\ \frac{dx+ey+f}{gx+hy+k} \\ 1 \end{bmatrix} = \begin{bmatrix} \hat{x}/\hat{w} \\ \hat{y}/\hat{w} \\ 1 \end{bmatrix} \sim \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$r_{x_i}(H) = ax_i + by_i + c - x_i'(gx_i + hy_i + k)$$

$$r_{y_i}(H) = dx_i + ey_i + f - y'(gx_i + hy_i + k)$$

$$A\vec{x} - \vec{b}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1'x_1 & -x_1'y_1 & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1'x_1 & -y_1'y_1 & -y_1' \\ & & & & \vdots & & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ k \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vec{0} \end{bmatrix}$$

$$\min_T \|A\vec{x}\|^2 \qquad \vec{x} = \vec{0} \qquad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

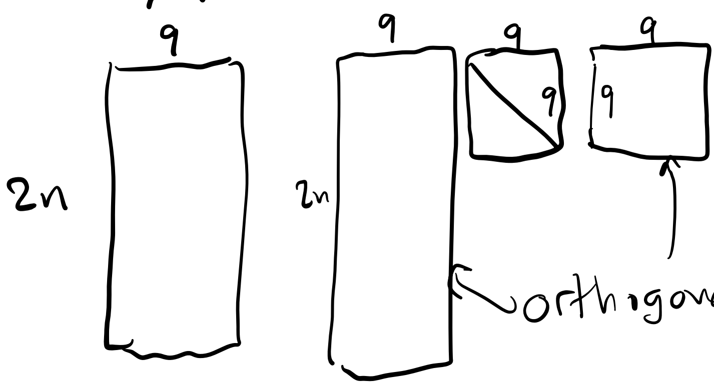
S.T. $\|\vec{x}\| = 1$

Subject To

$$\min \|A\vec{x}\|^2 : \boxed{x^T A^T A \vec{x}}$$

Singular Value Decomposition

$$A = U \Sigma V^T$$



orthogonal: $U^T U = I$

$$\vec{x}^T (V \Sigma U^T)(U \Sigma V^T)\vec{x}$$

$$\vec{x}^T V \Sigma \boxed{\Sigma V^T \vec{x}}$$

$$\Sigma^2$$

$$\begin{pmatrix}\sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_9\end{pmatrix}\begin{bmatrix} — v_1 — \\ — v_2 — \\ \vdots \\ \boxed{v_9}\end{bmatrix}\begin{bmatrix}\vec{x}\end{bmatrix}$$

math

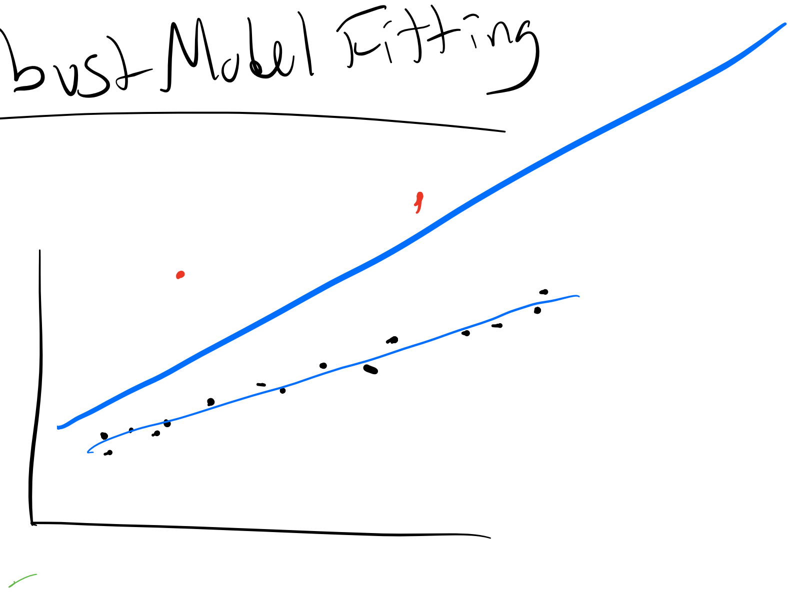
TL;DM

$$\begin{bmatrix} \sigma_1 v_1 \vec{x} \\ \sigma_2 v_2 \vec{x} \\ \vdots \\ \sigma_q v_q \vec{x} \end{bmatrix}$$

1. Compute SVD of A
2. Find index of smallest σ_i
3. Take corresponding row of V^T
   (col of V)

   set that to h.

Robust Model Fitting



RANSAC: RANdom Sample Consensus

For $i = 1$ to $\boxed{K}$:

$\quad d_i \leftarrow$ random sample of $\boxed{S}$ points

$\quad M_i \leftarrow$ fit model to d_i

$\quad$ inlier_count $\leftarrow \sum \mathbb{1}(|M(x_i) - y_i| < \boxed{\delta})$

$\quad$ if inl_count > best_count

$\quad\quad$ best_model $\leftarrow M_i$

$\quad\quad$ best_inliers $\leftarrow \{(x_i, y_i) : |M_i(x_i) - y_i| < \delta\}$

$\quad\quad$ best_count $\leftarrow$ inlier_count

$M_{final} \leftarrow$ fit model to best_inliers

k: # iterations

s: minimal set size ($\approx$ # DOF)

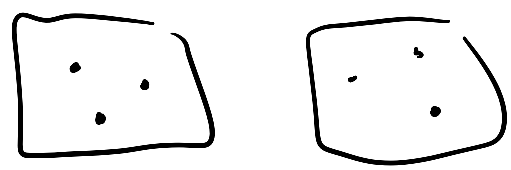
δ: inlier threshold

δ: noise model

s: line fitting: 2

translation: 1 pair

affine: 3 pairs



homography: 4 pairs

k: Suppose we want s inliers w/ probability $\geq P$ knowing that $r = \frac{\text{\# inliers}}{\text{\# total pts}}$ is the inlier ratio.

$$P(\text{choose all inliers}) = r^s$$

$$P(\text{at least one outlier}) = 1 - r^s$$

↑ bad thing

$$P(\text{all at least one outlier after } k \text{ trials}) = (1 - r^s)^k$$

↑ ~~bad~~ thing k times

What K do I need for $P(\text{success}) \geq P$

$$P \leq 1-(1-r^s)^K$$

$$P = 1-(1-r^s)^K$$

$$1-P = (1-r^s)^K$$

$$\frac{\log(1-P)}{\log(1-r^s)} \geq K \cancel{\log(1-r^s)}$$