Lecture 8: Projective Transformations

Alignment: Translation and Affine

Homogeneous Coordinates:

A $\overset{n+1}{3}$-vector representing a $\overset{n}{2}$D point

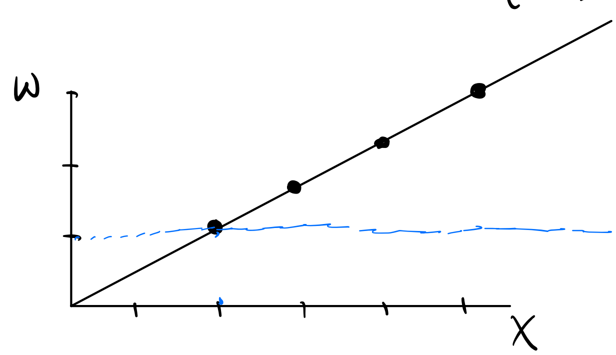
To get the "true" coordinates back, we need to normalize.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \xrightarrow{M} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix} \sim \begin{bmatrix} x'/w \\ y'/w \\ 1 \end{bmatrix}$$

Geometric Intuition

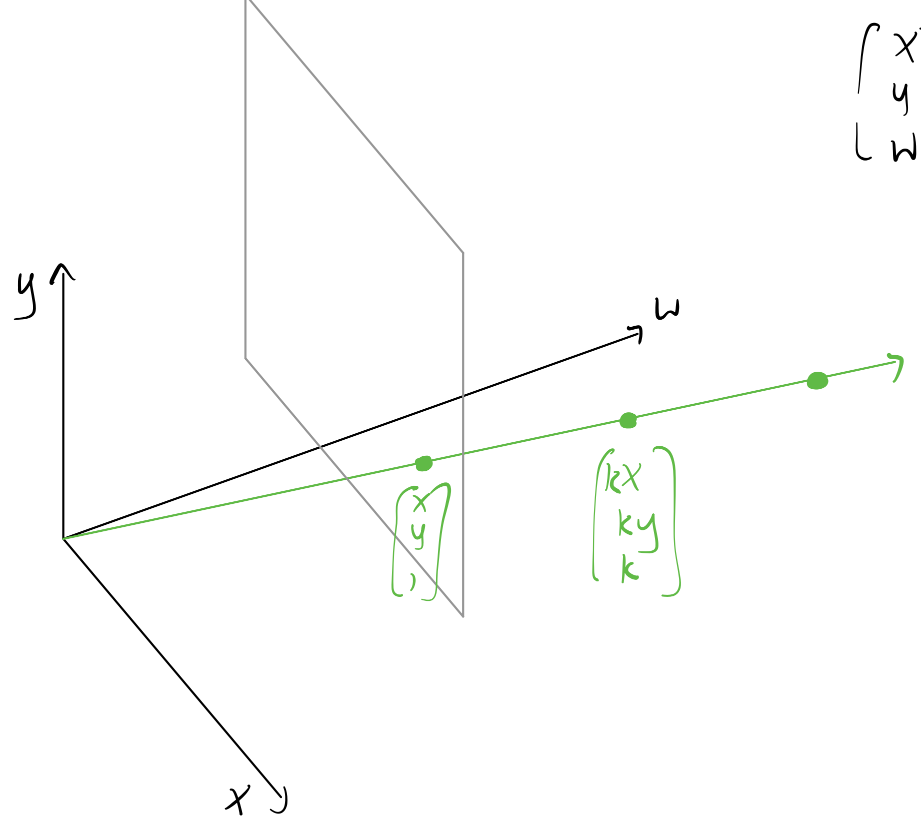
$$\begin{bmatrix} x \\ w \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

1D:



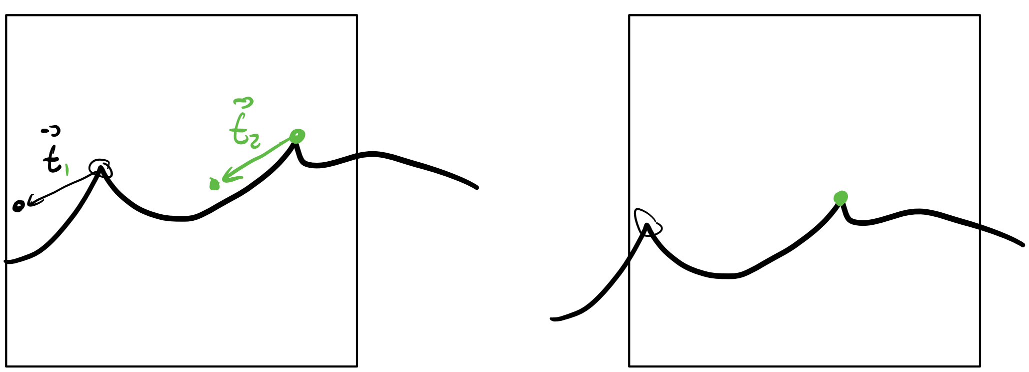
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

2D:



Translation

$$\begin{bmatrix} I_{2\times2} & \vec{t} \\ 0 \quad 0 & 1 \end{bmatrix}$$



One match:

$$x_i' = x_i + x_t$$

$$y_i' = y_i + y_t$$

More matches:

$$X_t = \frac{1}{n}\sum (x_i' - x_i) \qquad y_t = \frac{1}{n}\sum (y_i' - y_i)$$

More matches, but more math too:

$$r_{x_i}(X_t) = x_i' - (x_i + X_t)$$

$$r_{y_i}(y_t) = y_i' - (y_i + y_t)$$

$$A\vec{x} - b$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ \vdots & \end{bmatrix} \begin{bmatrix} X_t \\ y_t \end{bmatrix} - \begin{bmatrix} x_1' - x_1 \\ y_1' - y_1 \\ x_2' - x_2 \\ \vdots \end{bmatrix}$$

$$\left\| \begin{bmatrix} X_t - (x_1' - x_1) \\ y_t - (y_1' - y_1) \\ \vdots \end{bmatrix} \right\| \qquad \|Ax - b\|$$

Affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \quad \begin{bmatrix} \hat{x}' \\ \hat{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Residuals:

where the transform sent x to

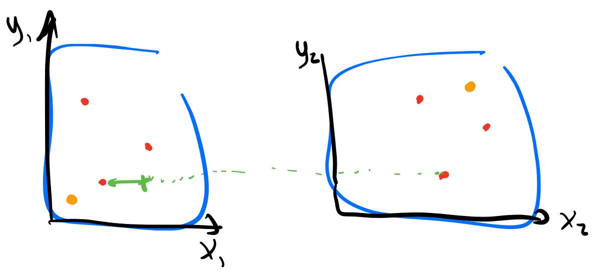
$$r_{x_i} = \boxed{(a x_i + b y_i + c)} - x'$$

$$r_y = \boxed{(dx + ey + f)} - y'$$

$$A \quad x \quad - \quad b$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & \vdots & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} - \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ \vdots \end{bmatrix}$$

$$T^* \quad \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$



y_1
x_1
y_2
x_2