Lecture 6 - Linear & Affine Transformations
Feature Descriptors

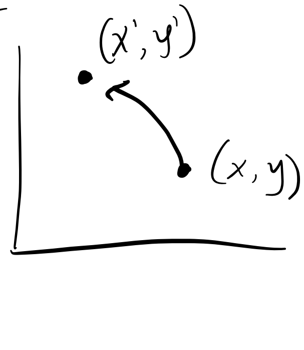
Matrix-vector product:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} — & \vec{r}_1 & — \\ & \vdots & \\ — & \vec{r}_i & — \\ & \vdots & \\ — & \vec{r}_m & — \end{bmatrix} \begin{bmatrix} | \\ \vec{x} \\ | \end{bmatrix} \qquad y_i = r_i \cdot \vec{x}$$

$$\begin{bmatrix} \\ \vec{y} \\ \\ \end{bmatrix} = \begin{bmatrix} | & & | & & | \\ c_1 & \cdots & c_i & \cdots & c_n \\ | & & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} \qquad \vec{y} = x_1 \vec{c}_1 + \cdots + x_i \vec{c}_i + \cdots + x_n \vec{c}_n$$

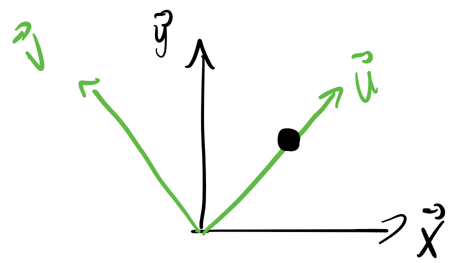
Mapping: $\mathbb{R}^2 \to \mathbb{R}^2$

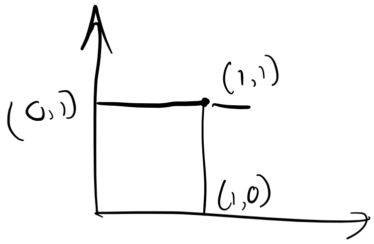
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

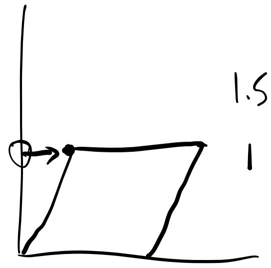


Change of Basis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{pmatrix} | & | \\ \vec{u} & \vec{v} \\ | & | \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$







$$\begin{matrix}1.5\\1\end{matrix} = \begin{bmatrix}1 & 0.5\\0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}0.5 & 0\\0 & 2\end{bmatrix}$$

Translation: Impossible!

Soln 1: $(M, \vec{t})$

$$T_2(T_1(\vec{x})) = M_2(M_1\vec{x} + \vec{t}_1) + t_2$$

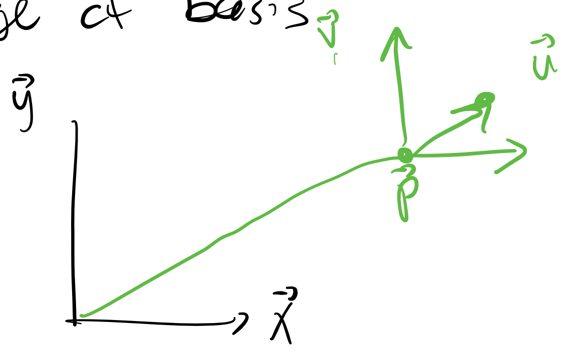
$$M_2 M_1$$

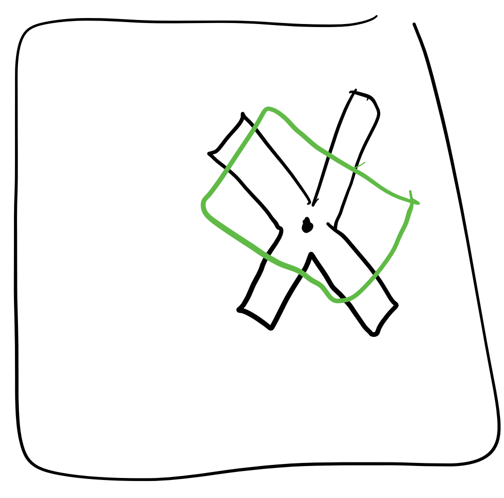
Soln 2: <u>Hack</u>

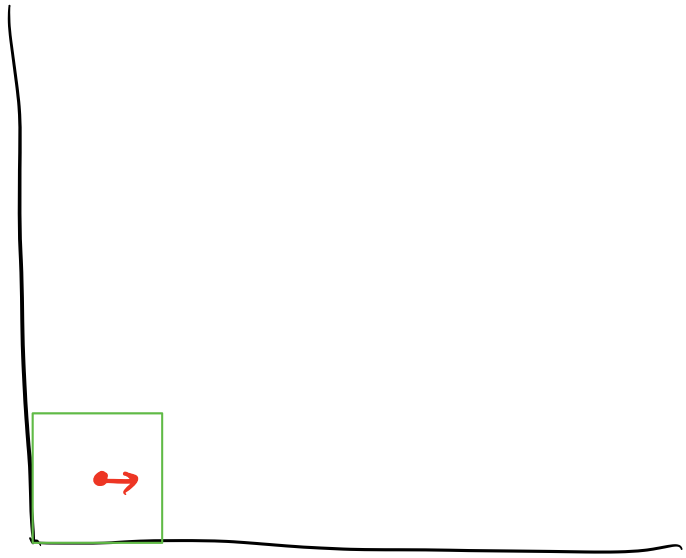
$$\begin{bmatrix}x+t_x\\y+t_y\\1\end{bmatrix} = \begin{bmatrix}1 & 0 & t_x\\0 & 1 & t_y\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$ homogeneous coordinates

affine transformation

Change of basis







1. Move to origin
2. Scale uniformly by $\frac{1}{8}$
3. Find gradient angle θ, rotate by $-\theta$
4. Shift by $(+2.5, +2.5)$