CS401 - Problem Set 8

1. (a) Without oracles, we proved $\mathbf{NP} \subseteq \mathbf{PSPACE}$ using simulation. This is because, in our proof, the **PSPACE** machine simulates what the **NP** machine does. Show that the proof still works for oracular machines. That is, prove that for every language O, $\mathbf{NP}^{\mathsf{O}} \subseteq \mathbf{PSPACE}^{\mathsf{O}}$.

   (b) The example from part (a) should help you see why simulation proofs "relativize." Please explain more generally why, if you prove $\mathbf{A} \subseteq \mathbf{B}$ using a simulation, then for any oracle O, $\mathbf{A}^{\mathsf{O}} \subseteq \mathbf{B}^{\mathsf{O}}$? (This need not be a formal proof - try to make your explanation as simple as possible.)

2. (a) Prove (or sketch a proof) that for every language O, $\mathbf{DTIME}(n)^{\mathsf{O}} \subseteq \mathbf{DTIME}(n^{1.5})^{\mathsf{O}}$.

   (b) The example from part (a) should help you see why diagonalization proofs "relativize." Please explain more generally why, if you prove $\mathbf{A} \subseteq \mathbf{B}$ using a diagonalization, then for any oracle O, $\mathbf{A}^{\mathsf{O}} \subseteq \mathbf{B}^{\mathsf{O}}$? (This need not be a formal proof - try to make your explanation as simple as possible. Think about what properties of TMs were needed for diagonalization, and argue those properties are still present for oracular TMs.)

3. [**Problem from Class**] Prove $\mathbf{coNP} \subseteq \mathbf{P}^{\mathsf{SAT}}$, where recall SAT is the language of Boolean formulas that have a satisfying assignment.