CS401 - Problem Set 7

1. Prove that SCDG is **NL**-complete, where

$$\mathsf{SCDG} = \{G : G \text{ is a strongly connected directed graph}\}. \tag{1}$$

   Strongly connected means that $\forall a, b \in V(G)$, if $a \neq b$, there is a path from a to b and a path from b to a. You can assume G is given as an adjacency matrix, where the (v, u)th bit of G is 1 if there is a directed edge from v to u, and 0 if there is no edge.

2. When we are proving a language is **NL**-Hard, we use a log-space reduction. However, when proving a language is **PSPACE**-hard, we showed we could not use a poly-space reduction. Why is it ok to use a log-space reduction in this case?

3. In the following problem, you should use the fact that there exists a universal space TM simulator $\mathcal{U}_S$ with the following properties. If $\alpha \in \{0,1\}^*$ describes a TM M_α and $M_\alpha(x)$ halts before using more than t cells of its work space, then $\mathcal{U}_S(\alpha, x, t) = M_\alpha(x)$ and $\mathcal{U}_S(\alpha, x, t)$ uses $C_{M_\alpha} t$ cells of its work space, where $C_{(M_\alpha)}$ is a constant that depends only on M_α. (That is, given α and α' such that $M_\alpha = M_{\alpha'}$ then $C_{(M_\alpha)} = C_{(M_{\alpha'})}$.) If $M_\alpha(x)$ does not halt before using t cells of its work space, then $\mathcal{U}_S(\alpha, x, t)$ uses at most Ct cells of its work space and outputs 0. Prove that $\mathbf{SPACE}(n) \subsetneq \mathbf{SPACE}(n^{1.5})$.

4. **Bonus Practice:** Consider a game Graph Explorer where on a graph G with a designated starting vertex s, where the first player chooses a vertex v connected to s, the other player chooses a vertex u connected to v, and the players continue alternating. The restriction is that you are no allowed to choose a vertex that has already been chosen. When a player is unable to make another move, that player loses. Consider the language

$$L = \{\langle G, s\rangle \text{ Player 1 has a winning Graph Explorer strategy on the graph } G \text{ starting at vertex } b. \tag{2}$$

   Prove Graph Explorer is in **PSPACE**