CS401 - Problem Set 5

CW: No proof write up. Work on you papers!

1. Crumbling Graph is a game played by two players on a directed graph $G = (V, E)$ with one vertex labelled as the starting vertex. Player 1 starts with a token at the starting vertex. Then Player 1 and Player 2 alternate taking turns, where Player 1 can moved their token from their current vertex along one outgoing edge to a new vertex. Player 2 can remove one outgoing edge from the vertex where Player 1's token currently sits. The game ends when Player 1 can no longer move. Let L be the following language:

$$L = \{\langle G, s, T\rangle : \text{Player 1 can make at least } T \text{ moves in the Crumbling Graph} \quad (1)$$
$$\text{game on the graph } G \text{ with starting vertex } s\}. \quad (2)$$

   Prove $L \in$ **PSPACE**. (For super duper extra learning, prove that $L \in$ **PSPACE**-Hard via a reduction from TQBF, or from another known **PSPACE**-Hard language. You will have to look up papers or other course notes to learn about techniques for doing this.)

2. Prove **NP** $\subseteq$**PSPACE**.