CS401 - Problem Set 4

1. Understanding why **NP** $\subseteq$ **EXP** will not help you to stop climate change, will not help you to come up with a cure for COVID, will not help you start a company that employs lots of people, and will not prevent wars. In fact, **NP** $\subseteq$ **EXP** has almost no bearing on anything in our physical world, given that we don't believe interesting problems in these classes can be solved by computers in any reasonable time. **NP** itself is not even physical; nondeterministic computation doesn't exist!!

   Given these facts, why are we spending 12 weeks of our lives thinking about these types of problems? What is the point? Is it even ethical to be studying this topic, given the pressing problems facing our society and planet? We will discuss these questions in class. To prepare, think about the following:

   - Why are you taking this class? (I asked you this on the first day, but I suspect you didn't quite know what you were getting into at that point. Perhaps your feelings have changed now.)
   - To get a sense of various ways of addressing this issue, take a look at this discussion, through the posts that have at least 6 upvotes. Note those arguments that you find most compelling (in either direction).
   - The discussion on StackExchange is about mathematics generally - what about computational complexity in particular? Do the same arguments apply?

2. * Let

   $\mathsf{QUADEQ} = \{\langle x \rangle : x \text{ is a set of binary quadratic equations modulo 2 with a satisfying assignment}\}$.

   Examples of binary quadratic equations modulo 2 are:

   $$u_1u_1 + u_2u_3 = 0 \pmod 2 \tag{1}$$
   $$u_1u_4 + u_2u_5 + u_2u_4 = 1 \pmod 2 \tag{2}$$

   where each u_i must equal 0 or 1.

   Prove that $\mathsf{QUADEQ} \in$ **NP**-Hard.

3. Prove that the language HALT is **NP**-HARD but not **NP**-Complete, where

   $$\mathsf{HALT} = \{\langle \alpha, x \rangle : \alpha \text{ describes the TM } M_\alpha, \text{ and } M_\alpha \text{ halts on input } x\}. \tag{3}$$

   See Theorem 1.11 (book), §1.4.1 (online) for more on HALT, and last page for a hint.

4. (a) Prove that if **P** = **NP**, then **NP** = **coNP**.
   (b) Please restate the statement from part (a) using non-technical language.
   (c) What does this result tell us about the difference between finding a solution and checking all solutions. Is this surprising?