CS401 - Problem Set 10 - the last one!

1. What class ($\mathbf{\Sigma}_2$ or $\mathbf{\Pi}_2$) is the following language in:

$$L = \{\phi : \text{ there is exactly one satisfying assignment to the Boolean formula } \phi\}. \tag{1}$$

2. The class **DP** is the set of languages L for which there exist two languages $L_1 \in$ **NP** and $L_2 \in$ **coNP** such that $L = L_1 \cap L_2$. Let

$$\text{EXACT INDSET} = \{\langle G, k\rangle : \text{ the largest set of vertices where no vertex in the set has an edge to any other vertex in the set has size } k\}. \tag{2}$$

   Prove

   (a) EXACT INDSET $\in \mathbf{\Pi}_2^p$

   (b) EXACT INDSET $\in$ **DP**

   (c) Prove **DP** $\subseteq \mathbf{\Pi}_2^p$.

3. In class, to prove that **BPP** $\in \mathbf{\Sigma}_2 \cap \mathbf{\Pi}_2$, we only prove that **BPP** $\in \mathbf{\Sigma}_2$. We said that this implies the main result because **BPP** = **coBPP**, which you prove in Quiz 9. Use the facts that **BPP** = **coBPP** and **BPP** $\subseteq \mathbf{\Sigma}_2$ to prove **BPP** $\subseteq \mathbf{\Sigma}_2 \cap \mathbf{\Pi}_2$.

4. Prove that if 3SAT$\leq_p \overline{\text{3SAT}}$, then **PH** = **NP**. (The **PH** collapses to **NP**).