

# CS401 - Problem Set 1

1. Please read over the syllabus at [go/cs401](#) (you do not need to follow any links at this time, unless you are interested), and post at least one question, concern, or comment at this [Canvas Discussion](#). Please also “like” any other posts you agree with.
2. Follow [these instructions](#) to create your Learning Plan.
3. Many of you took the CRA computing survey in a CS class at the end of the Fall semester. If you did not take it in Fall 2021, please do so now: [survey link](#). (If you took it last academic year 2020-2021, but not yet this academic year, you should take it again.)
4. In this class we will usually describe what a TM (Turing Machine) does at a very high level using full English sentences (i.e. we might say, “the TM does a breadth-first search over the graph starting at vertex  $s$ ,” or “the TM checks the number of 1s in the string”) or we might describe it using high-level pseudocode. However, you should briefly remind yourself what a TM and a NTM (non-deterministic Turing Machine) are at a low level, including transition functions, configurations, etc. You can read about them in the text (Perusall link left-side banner on [Canvas site](#)): §1.2 and 2.1.2 (first paragraph).
5. In this problem, you will design proof recipes that you can follow for many of the proofs in this class. For each of the following, write a proof recipe. As an example, here is a proof recipe for “If  $P$ , then  $Q$ ” using contrapositive:

If not  $Q$ , then [insert mathematical definition of (not  $Q$ ) here]. This implies [describe a consequence of (not  $Q$ ) that relates to (not  $P$ )...repeat as needed]. Therefore, not  $P$ .

- (a) The language  $L$  is in the class  $\mathbf{C}$ , where  $\mathbf{C}$  is the set of languages that can be decided by TMs with property  $A$ .
  - (b) If  $\mathbf{C}_1$  is the set of languages that can be decided by TMs with property  $A$ , and  $\mathbf{C}_2$  is the set of languages that can be decided by TMs with property  $B$ , then  $\mathbf{C}_1 \subseteq \mathbf{C}_2$
  - (c) If  $\mathbf{C}_1$  is the set of languages that can be decided by a TM with properties  $A$  and  $\mathbf{C}_2$  is the set of languages that can be decided by a TM with properties  $B$ , then  $\mathbf{C}_1 = \mathbf{C}_2$ .
6. What is the decision problem associated with the following language? (See final page for some notation information.) Please describe the problem in as natural non-mathy terms as possible.

$$\{\langle p, q, k \rangle : (\exists n : p, q \in \{0, 1\}^n) \wedge (\exists S \subseteq [n] : (|S| \geq k) \wedge (\forall i \in S, p_i = q_i))\}. \quad (1)$$

7. The problem “ $k$ -Clique” takes as input an undirected graph  $G$  and decides if there is a set of at least  $k$  vertices where there is an edge between each pair of vertices in the set. Write the language associated with this problem. (Please use set builder notation, and as much math notation as possible.)
8. A common function problem is to find the shortest path between two vertices  $s$  and  $t$  in a graph  $G$ . This is not a decision problem. Please define a language that represents a decision version of this problem. (You can use English or math or both to describe the language. There is more than one correct answer.)

Notation:

- $[n] = \{1, 2, 3, \dots, n\}$
- $\{0, 1\}^n$  is the set of  $n$ -bit binary strings.
- If  $x \in \{0, 1\}^n$ , then for  $i \in [n]$ ,  $x_i$  is the value of the  $i$ th bit of  $x$ .