9s Probabilistic Computation Wednesday, April 20, 2022 1:19 PM Recall Non-deterministic Polynomial Time Computation: NTM M e greject

M(x,u) = reject Nondeforministic Moice H- X ----) X · · · polynomial time u a path through NTM M(x,u) the output of following u · M accepts if J u: M(x,u) accepts · M rejects if Y u: M(x,u) rejects Goals: Improve understanding of oracle proof. Understand where probabilistic computation fits in Probabilistic TM

ATAM M Polynomial Steps probabilistic · U is sequence of nondeterministic choices · M(x,u) is output of following u Maccepts X if Pr [M(x,u) = accept] = 2/3 M rejects X if Pr [M(X,u)=rejects] = 1/3 L BPP (Bounded Probabilistic Polynomial Time) LEBPP if I a probabilistic TM M, M should halt in polynomral time regardless of its random choices and $\forall x \in \{0,15\}^*$. If $x \in L \rightarrow M$ accepts «If X&L >> M rejects XEL $Pr[M(x,u)=accept] \ge \frac{1}{2} + [E]$ | bounded away $x \notin L$ $Pr[M(x,u)=reject] \ge \frac{1}{2} + [E]$ from 1/2Why 2/3? Mx 8. Repeatedly run M O(k) times 8 amplifying success
Nx 1. Take a majority vote If $x \in L$ $Pr[M(x,u) = accept] \ge 1 - O(\frac{1}{2^2})$ If $x \notin L$ $Pr[M(x,u) = reject) \ge 1 - O(\frac{1}{2^2})$ RP (Randomized Polynomial Time) LERP if I a probabilistic TM M, M should halt in polynomral time regardless of its random choices and · If XEL, R(M(X,u) = accepts] = 2/3 (accept) reject off x & L, Pr[M(X,N)=rejects]=1 reject · If M decides LERP, and M(x,u) accepts for a sequence of random choices u, What do you know? A) XEL B) X & L C) Nothing · If M decides LERP, and M(x,u) rejects for a sequence of random choices u, What do you know? C) Nothing A) X6L B) X#L L= 2 x e 20,13*: X contains at least one "1" } Let M be the PTM that looks at Zn bits of X, chosen at random, and accept it finds a "1" XEL > Pr[M accepts] = 2/3 X EL >> Pr [M rejects] = 1 Draw Map + Explain: P, BPP, RP, NP, PSPACE Strongly believe PSPACE BPP P=BPP 6 PERP Random choices gaccept reject - Then if XEL, Yu, M(X,u) accepts, so Pr [M(x,u) accepts] = 2/3 - Then if X & L V u M(x,u) rejects, so Pr (M(x,u) rejects] = 1 · RP = BPP Use same machine, it will satisfy BPP definition RP (Randomized Polynomial Time) LERP if] a probabilistic TM M s.t. M halfs in polynomial time regardless of its random choices, and $\forall x \in \{0,1\}^n$ ·If XEL Pr[M(X,u) = accept] = 2/3 · If X&L Pr[M(x,u)=accept) = 0 = 1/3 allowed for BPP machme · RP SNP > Turn probabilistic choices into nondeterministic choices. erasing workspace in between ? BPP = NP? PSPACE machine: iterate through every path, · BPP = PSPACE and keep a running total of # of accepts Sum: 2 It accepts if at loas 2/3 of runs accept.