Goals:

- Prove tight(ish) relationship between SPACE and NSPACE
- Think about LOG-space classes

Announcements:

- Reflection 3
- Quiz 5, Paper 1 tomorrow
- Tapia Panel: Monday on Zoom @ 7pm.
- How to replace u1 with u1i?

PSPACE = NPSPACE

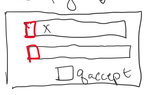
$NSPACE(s) \subseteq SPACE(s^2)$? $d = 1000$?

$O(s) \rightarrow a \cdot s$

Savitch's Theorem

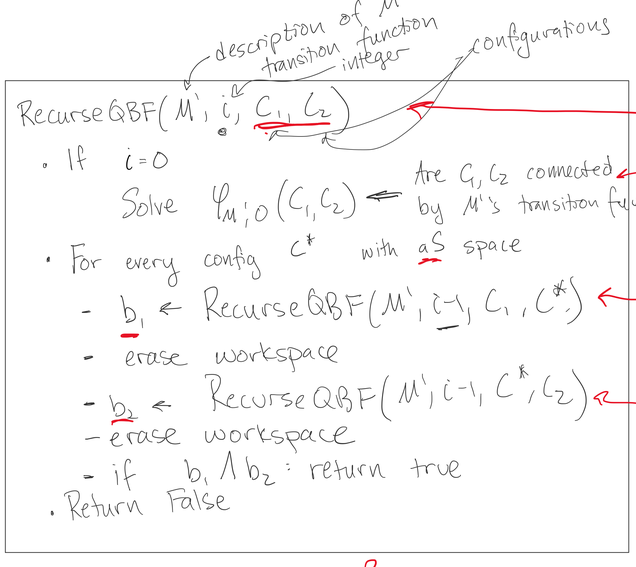
$NSPACE(s) \subseteq SPACE(s^2)$

Let $L \in NSPACE(s)$. Then $\exists\ a \in \mathbb{N}$, NTM M' that use at most as space and decides L, and has unique accepting config ↴



$C_{accept,x}$

Consider:



$R(i,s)$ is space used

$O(s)$ Write down

$O(s)$

$O(s)$

$R(i-1,s)$

$O(1)$

$R(i,s) = R(i-1,s) + O(s)$

$= R(i-2,s) + O(s) + O(s)$

$= R(i-3,s) + O(s) + O(s) \cdots O(s)$

$= R(0,s) + O(is)$

$= O(s) + O(is)$

$= O(i \cdot s)$

$O(O(s) \cdot s)$

$O(s^2)$

$O(s)$ at most $2^{O(s)}$ steps

1. What is space use?
2. What is RecurseQBF doing?

Checking if there is a config C^* halfway b/t C_1, C_2

⇓

Path from C_1 to C_2

Then RecurseQBF$(M', O(s), C_{start,x}, C_{accept,x})$ decides L in $O(s^2)$ space, so $L \in SPACE(s^2)$.

Log Space

Def: A $L \in$ L (logspace) if $\exists$ a TM M that decides L and uses $O(\log(n))$ space ($n = |x|$)

Def: A $L \in$ NL (nondeterministic logspace) ... NTM ...

★ $O(\log(n))$ not ~~$O((\log(n))^2)$~~ ★

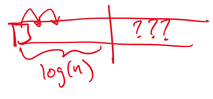
$\log(n^2)$

$2\log(n)$

Can't store the whole input in memory (R/W)!

For Discussion:

1. Why doesn't it make sense to discuss log-time classes?



2. L is one of the classes with the most real world relevance. What types of computation fit this model? Finding max elt. Internet, "Big Data", GPS, Internet of Things

3. PSPACE = NPSPACE, so does L = NL?

$NSPACE(\log(n)) = SPACE((\log(n))^2)$

NL ? L