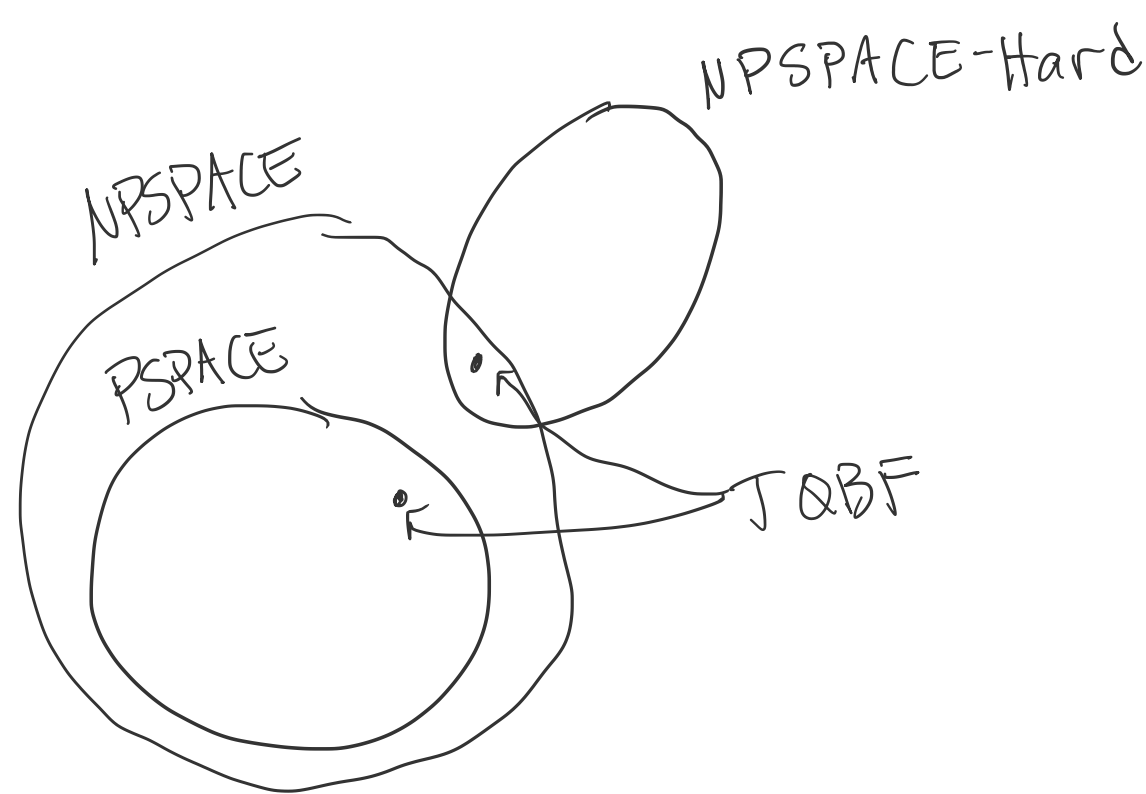


Runtime of $f_MakeQBF$?

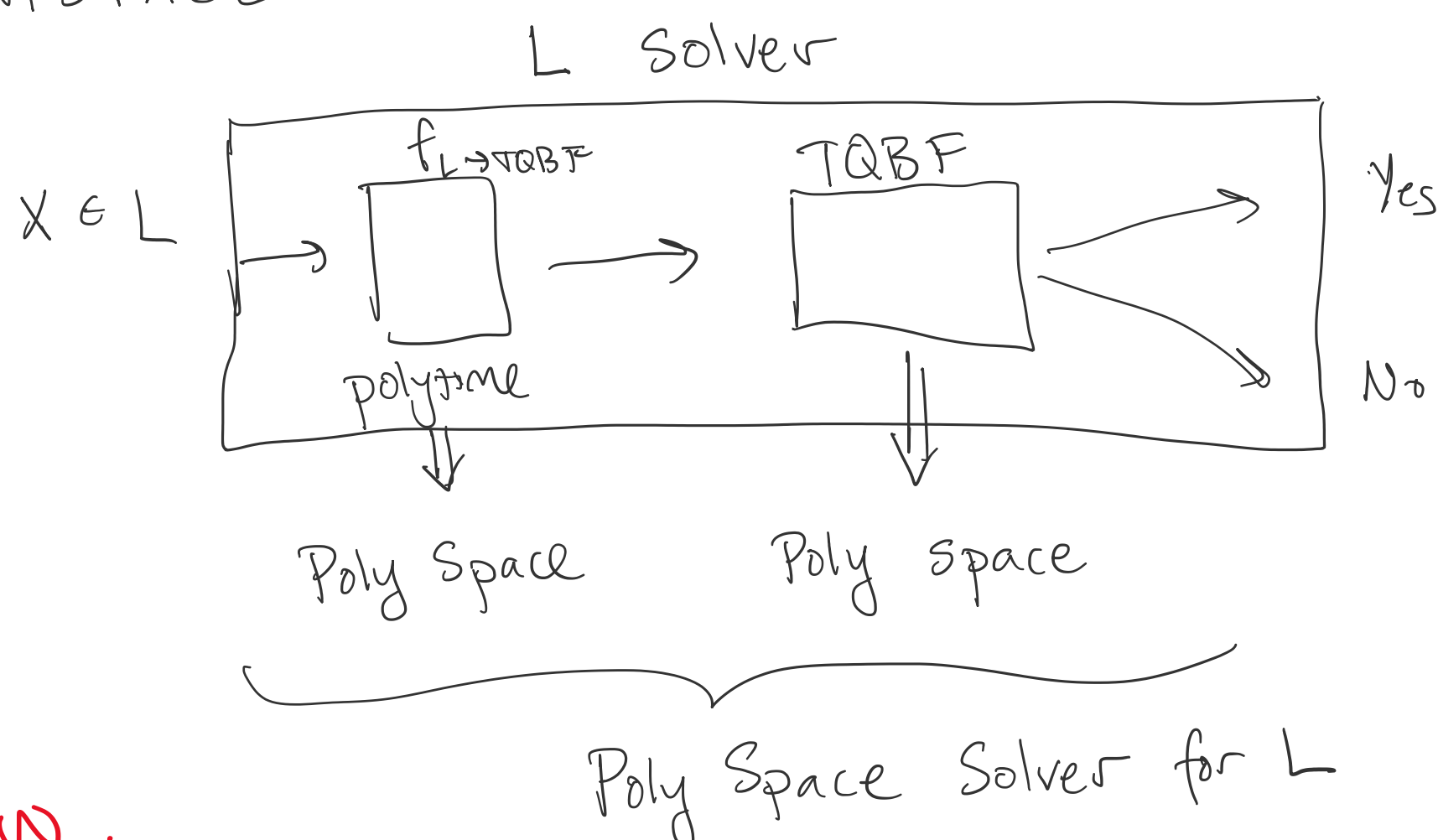
Big Picture?

How would we have to change proof to show TQBF is NPSPACE Hard?

" $\varphi_0(C_0, C_1)$ " now is a formula that checks if C_1 results from C_0 from either nondeterministic choice



$L \in NPSPACE \rightarrow$



We know something!

$L \in PSPACE$

Collapse!

Reductions Review (Proving $L^* \in \text{Class-Hard}$)

- Direct: $L \in \text{Class}$, $\exists$ a TM M w/ Property A , and can use transition function (tf) of M to create conversion function f from L input to L^* input s.t. $f(x, \text{tf}) \in L^*$ iff $x \in L$.

- Boot-Strap: Know $L^\heartsuit$ is Class-Hard.

Create conversion function f from $L^\heartsuit$ input to L^* input s.t. $f(x) \in L^*$ iff $x \in L^\heartsuit$

$3\text{-SAT}_3 = \{ \langle x \rangle : x \text{ describes a 3-SAT instance where each variable } u_i \text{ appears in at most 3 clauses} \}$

Prove 3-SAT_3 is NP-Hard

$$x = (u_{i1} \vee \neg u_{i2} \vee u_{i3}) \wedge (\neg u_{i2} \vee u_{i4} \vee u_{i5}) \wedge (u_{i3} \vee \neg u_{i2} \vee u_{i6}) \wedge (\neg u_{i4} \vee u_{i6} \vee \neg u_{i7}) \dots$$

$x \in 3\text{SAT}$ Don't figure out!

Create f to turn x into 3-SAT_3 instance

$$(u_{i1} \vee \neg u_{i2} \vee u_{i3}) \wedge (\neg u_{i1} \vee \neg u_{i4} \vee u_{i5})$$

$$(u_{i1} \vee \neg u_{i2}) \wedge (\neg u_{i1} \vee u_{i2})$$

- f : replace j^{th} occurrence of u_i with $u_{ij} \leftarrow \text{polytime}$ for each u_{ij} except last include clauses $\leftarrow \text{polytime}$

$$(u_{ij} \vee \neg u_{i(j+1)}) \wedge (\neg u_{ij} \vee u_{i(j+1)})$$

- If $x \in 3\text{SAT}$, $\exists (u_1, u_2, \dots, u_n)$, a satisfying assignment.

Then $u_{ij} = u_i$ will satisfy $f(x)$.

- If $f(x) \in 3\text{SAT}_3$, all u_{ij} must have same value.

Set u_i to have that value $\rightarrow$ satisfying assignment for x .