Hacker Thing, Do PS4 #1!

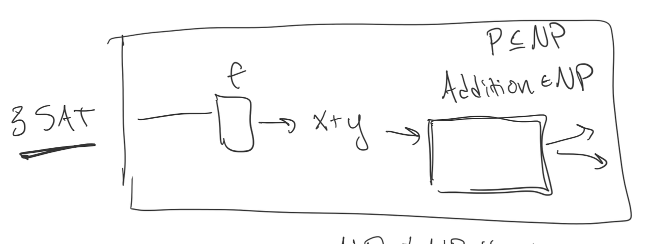
Goals

- Prove Nondeterministic Connection
- Build Understanding of coNP

Q's: how does a NTM work (IRL),

Q's: Everyone is kind of confused.

Q's: Why does NP-Hard not include all of NP.



$NP \neq NP\text{-Hard}$

Prove $NP_1 = NP_2$

$NP_1 \subseteq NP_2$, $NP_2 \subseteq NP_1$

$NP_1 \subseteq NP_2$

Let $L \in NP_1$. The [def]. Let M' be the NTM that acts like M, except whenever M reads a bit of u, M' makes a non-deterministic choice between what M would do if the bit of u were 0 or 1. Since M runs in polytime, M' runs in polytime. If $x \in L$, $\exists u$ s.t. $M(x,u)=1$, so the corresponding path in M' will accept.

Try to do on your own!

$NP_2 \subseteq NP_1$

Let $L \in NP_2$. Then [def]. Let M' be a TM that on input $\langle x, u \rangle$ follows the non-deterministic path of M given by u. Since M is polytime, M' is polytime. If $x \in L$, there is an accepting path, so there is a witness u s.t. $M(x,u)=1$. If $x \notin L$, there is no accepting path, so no witness will cause M' to accept. Thus $L \in NP_1$.

def:

Let $L \subseteq \{0,1\}^*$. Then $\bar{L} \subseteq \{0,1\}^*$, the complement of L, is $\bar{L} = \{x : x \notin L\}$.

$L = \{01, 10, 11, 00\}$

$\bar{L} = \{\phi, 1, 0, 000, 001, \ldots\}$

def

$coNP = \{L : \bar{L} \in NP\}$

$L = \{\langle x \rangle : x$ describes a sudoku grid with no solutions$\}$

Prove $L \in coNP$

$\Updownarrow$

Prove $\bar{L} \in NP$

$\bar{L} = \{\langle x \rangle : x$ describes a sudoku grid with a solution

OR

x does not describe a sudoku grid$\}$

Define M that acts on $\langle x, u \rangle$

↓

Checks

accepts
- x is not a sudoku grid
- x is a grid + u is a solution

What is wrong with the following proof that

coNP ⊆ NP? ??

Pf

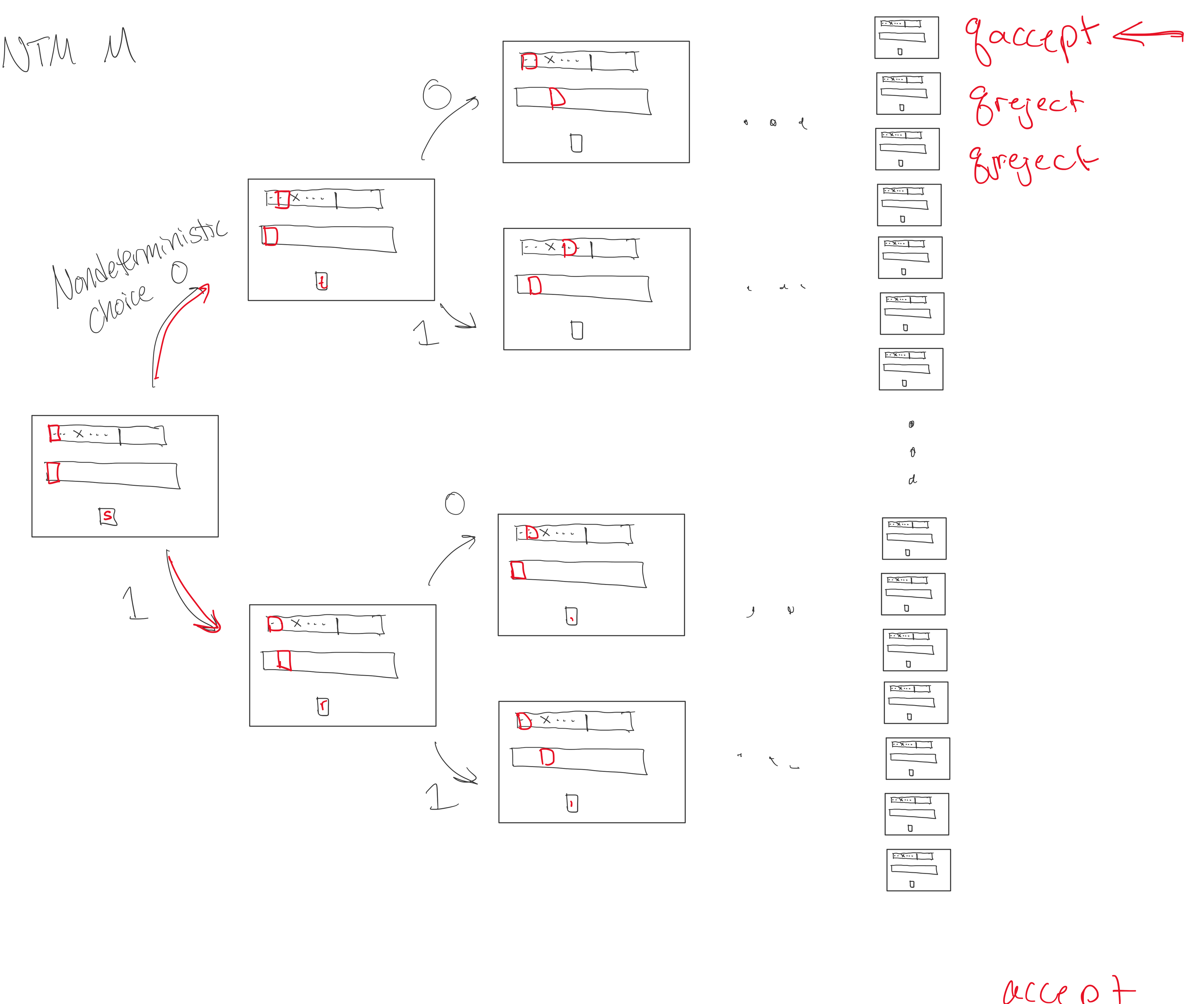
Let $L \in coNP$. Then $\bar{L} \in NP$. Then there exists a polytime NTM M s.t. $x \in \bar{L}$ iff $M(x)=1$. Then create a new NTM M' that on any path where M accepts, M' rejects, and any path where M rejects, M' accepts. Thus

False

$$x \notin \bar{L} \longleftrightarrow M(x)=0 \longrightarrow M'(x)=1 \longleftrightarrow x \in L$$

Thus M' decides L, and since M is a polynomial time NTM, we have $L \in NP$.

NTM M



NTM M'