

## Goals

- Understand NP Definition

NP

Let  $L \subseteq \{0,1\}^*$ . Then  $L \in \text{NP}$  if  $\exists$  a polynomial  $p: \mathbb{N} \rightarrow \mathbb{N}$  and a polytime TM  $M$  s.t.  $\forall x \in \{0,1\}^*$

$$x \in L \text{ iff } \exists u \in \{0,1\}^{p(|x|)} \text{ s.t. } M(x,u) = 1.$$

$\rightarrow$  If  $x \in L$ , then  $\exists u \in \{0,1\}^{p(|x|)}$  s.t.  $M(x,u) = 1$   
 $\rightarrow$  If  $x \notin L$ , then  $\forall u \in \{0,1\}^{p(|x|)}$ ,  $M(x,u) = 0$

Terminology:

- $M \Rightarrow$  Verifier
- $u \Rightarrow$  witness or certificate

$m \times m$  grid  $O(m^2)$

ex:

$L = \{ \langle x \rangle : x \text{ is a solvable sudoku grid} \}$

$u?$  fill of the rest of the grid

$M?$  checks each row + col