Goals

- Practice problem solving and proving with reduction
- Gain intuition about reductions

Candidate
Career talks
Quiz 1 (open all except general internet)
Tutoring?
Pset changes

Prove: $\boxed{\text{If } L' \in P \text{ and } L \leq_p L'}$ then $\boxed{L \in P}$

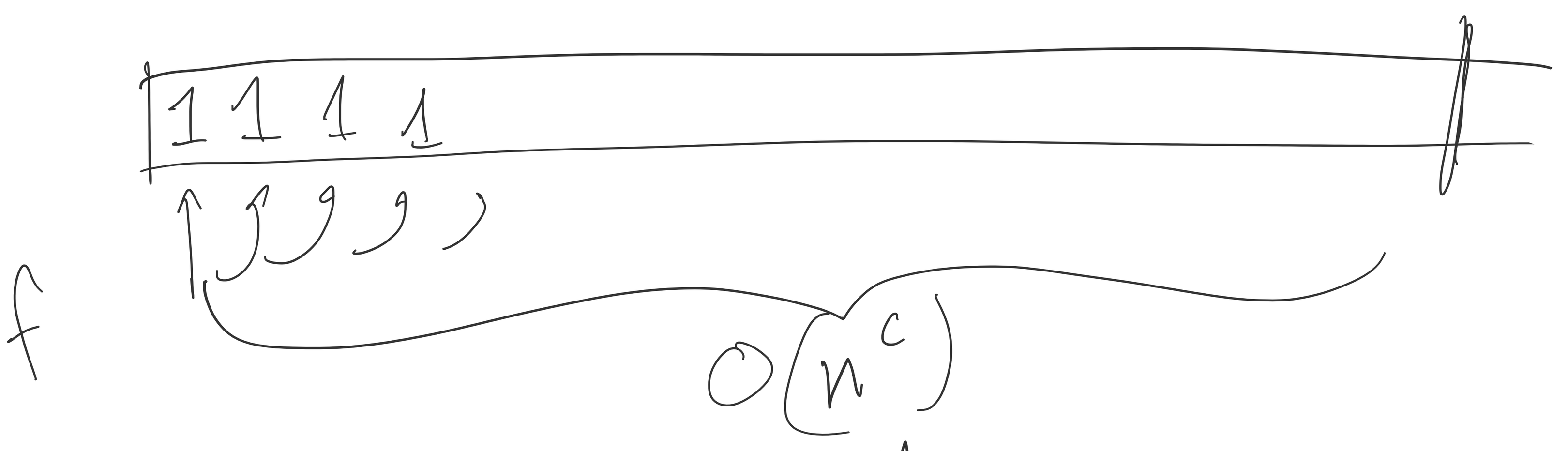
$\boxed{\text{Since } L' \in P \text{ there is a TM } M' \text{ that decides } L' \text{ in } O(n^c) \text{ time for some } c \in \mathbb{N}.}$

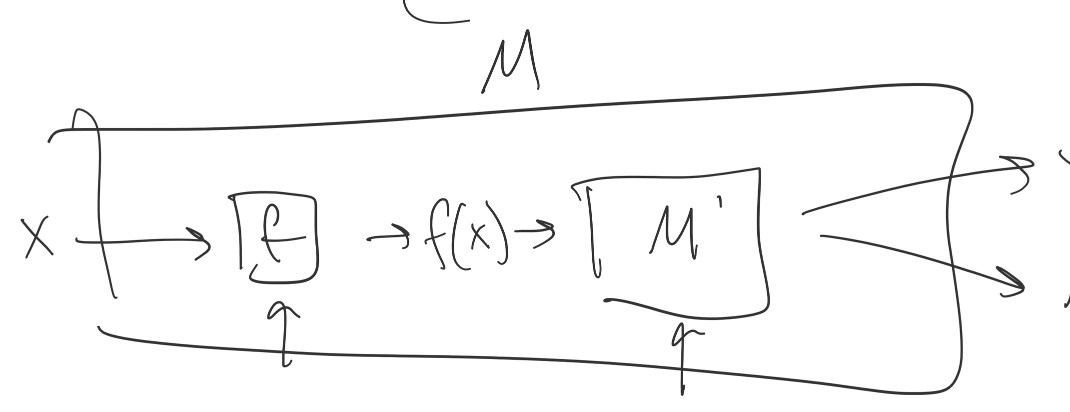
Since $L \leq_p L'$, there is an $O(n^d)$ time algorithm f for some $d \in \mathbb{N}$, such that $x \in L$ iff $f(x) \in L'$.

Let M be the TM that does [ ]
Then $M(x) = 1$ iff $x \in L$ b/c [ ]
Now M runs in $O(n^{??})$ steps b/c
[ ]

Let M be the TM that on input x, implements f to create $f(x)$, and then implements M' on input $f(x)$ and outputs $M'(f(x))$. Then $M(x) = 1$ iff $x \in L$ because $x \in L$ iff $f(x) \in L'$, and M' decides L'.

Now on input x, f uses $O(|x|^c)$ time and creates an output $f(x)$ that has length $O(|x|^c)$.
Then M' runnin on $f(x)$ uses $O((|x|^c)^d)$ time. This gives a total runtime of $O(n^{cd} \cancel{+ n^c})$. Thus $L \in P$.

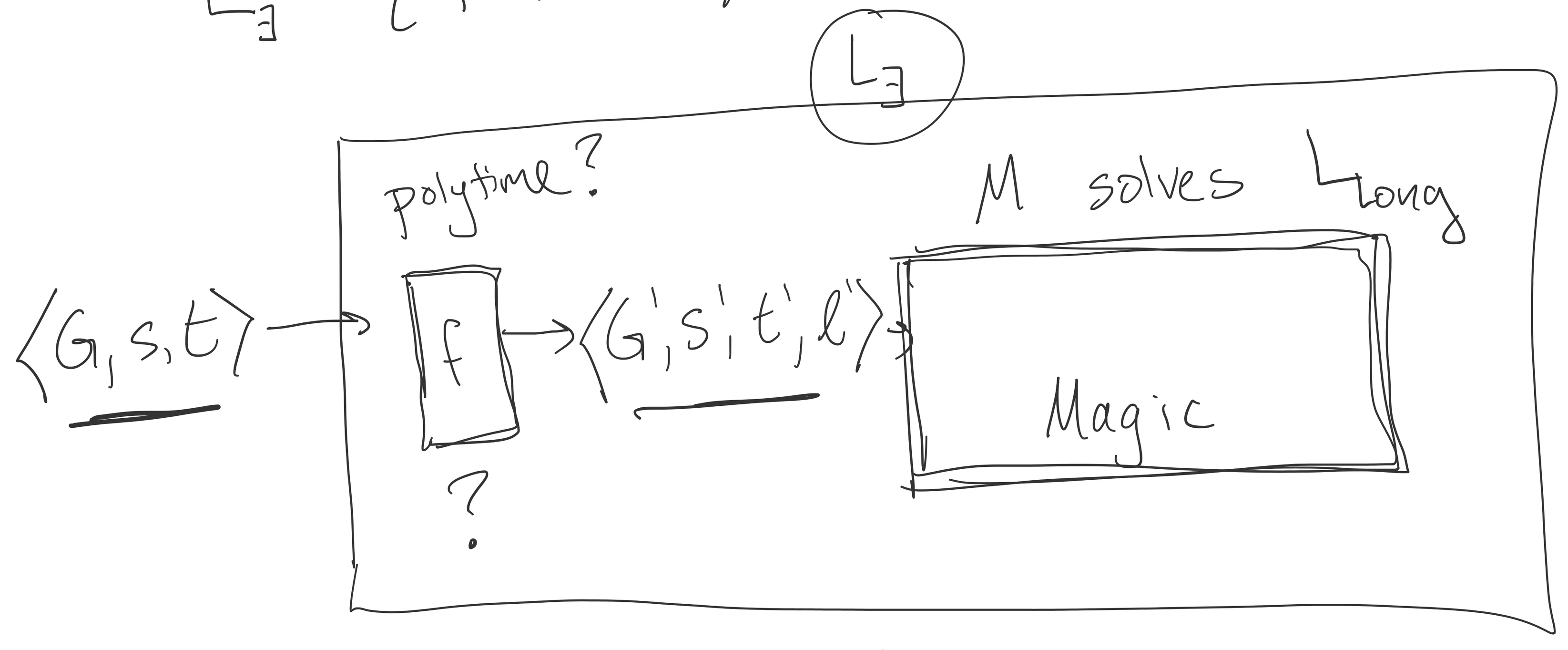




Reduction example

$L_{Long} = \{\langle G, s, t, \ell \rangle$: There is a path from s to t with length at least ℓ in graph $G\}$

$L_{\exists} = \{\langle G, s, t \rangle$: There is a path from s to t in $G\}$



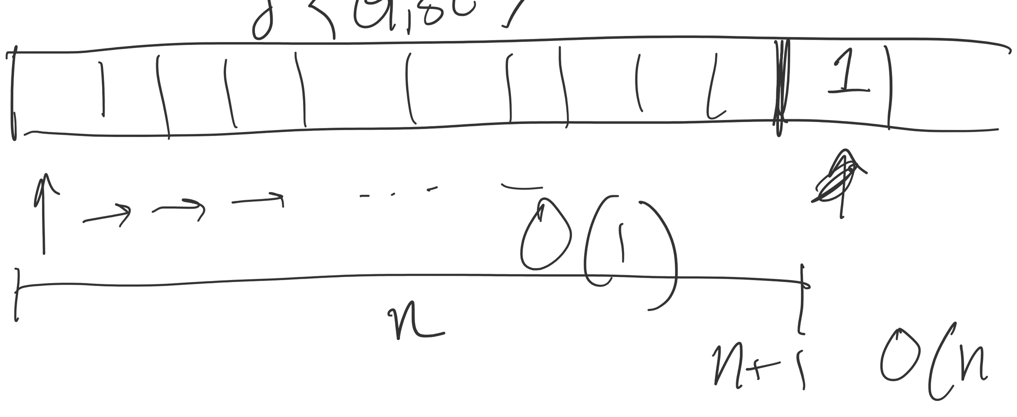
$L_\exists \in P$

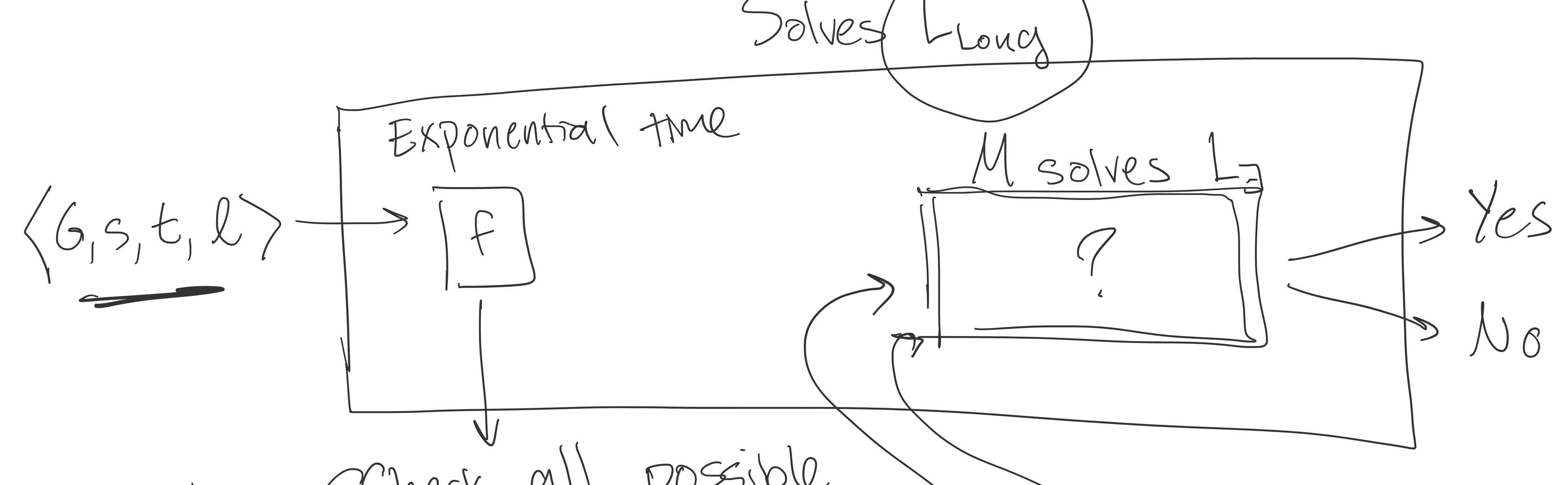
$L_\exists \leq_p L_{Long}$

$L_{Long} \in NP$

$\langle G, s, t \rangle \in L_\exists$ iff $\langle G', s', t', \ell' \rangle \in L_{Long}$

$\langle G, s, t \rangle \overset{f}{\Rightarrow} \langle G, s, t, 1 \rangle$





$(N-1)(N-2)\cdots$
$O(n^n)$

Check all possible paths from s to t.
If there is a path of length at least ℓ
$\langle \{s, t\}, s, t \rangle$

Else
$\langle \emptyset, s, t \rangle$