CS401 - Problem Set 9

1. What class ($\mathbf{\Sigma}_2$ or $\mathbf{\Pi}_2$) is the following language in:

$$L = \{\phi : \text{ there is exactly one solution to the Boolean formula } \phi\}. \tag{1}$$

Prove it!

2. The class **DP** is the set of languages L for which there exist two languages $L_1 \in$ **NP** and $L_2 \in$ **coNP** such that $L = L_1 \cap L_2$. Let

$$\textsf{EXACT INDSET} = \{\langle G, k\rangle : \text{ the largest set of vertices where no vertex in the set}$$
$$\text{has an edge to any other vertex in the set has size } k\}. \tag{2}$$

Prove

(a) $\textsf{EXACT INDSET} \in \mathbf{\Pi}_2^p$

(b) $\textsf{EXACT INDSET} \in$ **DP**

(c) Prove **DP** $\subseteq \mathbf{\Pi}_2^p$.

3. In class, to prove that **BPP** $\in \mathbf{\Sigma}_2 \cap \mathbf{\Pi}_2$, we only prove that **BPP** $\in \mathbf{\Sigma}_2$. We said that this implies the main result because **BPP** = **coBPP**. First prove that **BPP** = **coBPP**. Then prove that if **BPP** = **coBPP** and **BPP** $\in \mathbf{\Sigma}_2$, then **BPP** $\in \mathbf{\Sigma}_2 \cap \mathbf{\Pi}_2$.