

CS401 - Problem Set 8

CW: #4

1. (a) Prove that for every language $0 \in \{0, 1\}^*$, $\mathbf{NP}^O \subseteq \mathbf{PSPACE}^O$.
(b) Try to explain why simulation proofs “relativize.” In other words, if you prove $\mathbf{A} \subseteq \mathbf{B}$ using a simulation, as we typically do, why is it that for all oracles O , $\mathbf{A}^O \subseteq \mathbf{B}^O$? (This need not be a formal proof - try to make your explanation as simple as possible.)
2. (a) Prove (or sketch a proof) that for every language $O \in \{0, 1\}^*$, $\mathbf{DTIME}(n)^O \subsetneq \mathbf{DTIME}(n^{1.5})^O$.
(b) Try to explain why diagonalization proofs “relativize.” (As we’ve seen in our one example, diagonalization requires the ability to represent TMs using strings, and the existence of a universal simulator that can simulate any TM with low overhead. These two properties allow us to create a function f that takes as input a string that represents a TM, and make the output of f dependent on properties of the universal TM.)
3. [Problem from Class] Prove $\mathbf{coNP} \subseteq \mathbf{P}^{\mathbf{SAT}}$, where recall \mathbf{SAT} is the language of Boolean formulas that have a satisfying assignment.
4. Show that there is a language $B \in \mathbf{EXP}$ such that $\mathbf{NP}^B \neq \mathbf{P}^B$.