CS401 - Problem Set 7

CW: No. 3

1. Prove that SCDG is $\textbf{NL}$-complete, where

$$\mathsf{SCDG} = \{G : G \text{ is a strongly connected directed graph}\}. \tag{1}$$

   Strongly connected means that $\forall a, b \in V(G)$, if $a \neq b$, there is a path from a to b and a path from b to a. You can assume G is given as an adjacency matrix, where the (v, u)th bit of G is 1 if there is a directed edge from v to u, and 0 if there is no edge.

2. When we are proving a language is $\textbf{NL}$-Hard, we use a log-space reduction. However, when proving a language is $\textbf{PSPACE}$-hard, we showed we could not use a poly-space reduction. Why is it ok to use a log-space reduction in this case, especially in the context of Savitch's theorem?

3. In the following problem, you should use the fact that there exists a universal space TM simulator $\mathcal{U}_S$ with the following properties. If $\alpha \in \{0,1\}^*$ describes a TM M_α and $M_\alpha(x)$ halts before using more than t cells of its work space, then $\mathcal{U}_S(\alpha, x, t) = M_\alpha(x)$ and $\mathcal{U}_S(\alpha, x, t)$ uses $C_{M_\alpha} t$ cells of its work space, where $C_{(M_\alpha)}$ is a constant that depends only on M_α. (That is, given α and α' such that $M_\alpha = M_{\alpha'}$ then $C_{(M_\alpha)} = C_{(M_{\alpha'})}$.) If $M_\alpha(x)$ does not halt before using t cells of its work space, then $\mathcal{U}_S(\alpha, x, t)$ uses at most Ct cells of its work space and outputs 0. Prove that $\textbf{SPACE}(n) \subsetneq \textbf{SPACE}(n^{1.5})$.

4. [**Extra Practice**] Use the approach from class that we used to prove $\textbf{DTIME}(n) \subsetneq \textbf{DTIME}(n^{1.5})$ to prove that if f and g are functions such that $f(n) \log f(n) = o(g(n))$, then $\textbf{DTIME}(f(n)) \subsetneq \textbf{DTIME}(g(n))$.

   "Little-oh" notation is defined as follows: we say $f(n) = o(g(n))$ if for every $\epsilon > 0$, there exists a value n_ϵ such that for all $n > n_\epsilon$, $f(n) \leq \epsilon g(n)$.