CS401 - Problem Set 6

No CW proof this week. Work on your papers!!!

1. Hey! We've made it halfway through the semester! Congratulations!

   Please fill out this feedback form to let me know how things are going so far. I'm especially interested in how you've been experiencing the lack of grades, since this is a new format for me, but other feedback is also welcome!

2. Understanding why **NP** $\subseteq$ **PSPACE** will not help you to stop climate change, will not help you to come up with a cure for COVID, and will not help you start a company that employs lots of people. In fact, **NP** $\subseteq$ **PSPACE** has almost no bearing on anything in our physical world, given that we don't believe interesting problems in these classes can be solved by computers in any reasonable time. **NP** itself is not even physical; nondeterministic computation doesn't exist!! Given these facts, why are we spending 12 weeks of our lives thinking about these types of problems? What is the point? Is it even ethical to be studying this topic, given the pressing problems facing our society and planet? We will discuss these questions in class. To prepare, think about the following:

   - Why are you taking this class?
   - To get a sense of various ways of addressing this issue, take a look at this discussion, through the posts that have at least 6 upvotes. Note those arguments that you find most compelling (in either direction).
   - The discussion on StackExchange is about mathematics generally - what about computational complexity in particular? Do the same arguments apply?

3. Crumbling Graph is a game played by two players on a directed graph $G = (V, E)$ with one vertex labeled as the starting vertex. Player 1 starts with a token at the starting vertex. Then Player 1 and Player 2 alternate taking turns, where Player 1 can moved their token from their current vertex along one outgoing edge to a new vertex. Player 2 can remove one outgoing edge from the vertex where Player 1's token currently sits. The game ends when Player 1 can no longer move. Let L be the following language:

$$L = \{\langle G, s, k\rangle : \text{Player 1 can make at least } k \text{ moves in the Crumbling Graph} \quad (1)$$
$$\text{game on the graph } G \text{ with starting vertex } s\}. \quad (2)$$

   Prove $L \in$ **PSPACE**. (For super duper extra learning, prove that $L \in$ **PSPACE**-Hard via a reduction from TQBF, or from another known **PSPACE**-Hard language. You will have to look up papers or other course notes to learn about techniques for doing this.)

4. As in class, let $\varphi_i(C_1, C_2) = 1$ (true) if there is a path of length at most 2^i from configuration C_1 to configuration C_2 in the configuration graph of a TM, and 0 otherwise. Explain why

$$\begin{aligned}\varphi_i(C, C') &\equiv \exists C^* : \varphi_{i-1}(C, C^*) \wedge \varphi_{i-1}(C^*, C') \\ &\equiv \exists C^* \forall D, D' \left(\left((D = C) \wedge (D' = C^*) \right) \vee \left((D = C^*) \wedge (D' = C') \right) \right) \rightarrow \varphi_{i-1}(D, D'). \end{aligned} \tag{3}$$

5. Let $L \in$ **PSPACE**, and let M be a machine that decides L in polynomial space. Analyze the runtime of the function $\hat{g}(i, C, C')$ for the TM M that appears above these exit tickets, in terms of the size of the configurations C, C'. Given your analysis, argue that the reduction from L to **TQBF** is polytime.

6. Briefly explain all of the inclusions in the diagram in question 4. (There are really only a couple of key ideas that can be used to explain all of the relationships. You don't need to explain why **PSPACE**=**NPSPACE**.)