

CS401 - Problem Set 4

CW: You can choose either 2a or 3a for your proof.

1. In class, we discussed how the definition of **NP** involving a witness (perhaps) captures the concept of creativity in problem solving. Now that we've seen the NTM definition of **NP**, can you make a connection between creativity and our new definition?
2. (a) Define the class **coNP_a** as follows. Let $L \in \mathbf{coNP}_a$ if there exists a polynomial p and a polytime TM M such that

$$x \in L \leftrightarrow \forall u \in \{0, 1\}^* \text{ s.t. } |u| \leq p(|x|), M(x, u) = 1. \quad (1)$$

Prove that **coNP_a** = **coNP**. (Note that in the definition of **coNP**, we have $|u| = p(|x|)$, and in the definition of **coNP_a**, we have $|u| \leq p(|x|)$.)

Hint: The proofs in both directions are a little subtle, especially the **coNP_a** \subseteq **coNP** direction. If you are thinking that it is obvious, remember that you need to exactly satisfy the descriptions in the definitions of each class. Make sure that if x is not in the language, your TM does not accept it.

- (b) In part (a), we explore how much we can tweak the standard definition of **coNP** while still maintaining the original class. This is important because we'd like to know what tweaks cause the class to collapse to **P**, and make the problem easy to solve, or what tweaks cause the class to become even more difficult. In non-technical language, explain which aspect of the class **coNP** we examined in part (a), and how the result informs our understanding about what properties of the definition of **coNP** are important.
3. (a) Prove that if **P** = **NP**, then **NP** = **coNP**.
(b) Please restate the statement from part (a) using non-technical language.
(c) What does this result tell us about the difference between finding a solution and checking all solutions. Is this surprising?