CS401 - Problem Set 1

* Proof problem for CW

Heads up that for many of you, this problem set will likely not need the kind of deep thinking and puzzling that later problem sets will require.

1. Please read over the syllabus at go/cs401, and write down any questions, concerns, or comments that you have.

2. Please remind yourself from CS 301 what a TM (deterministic Turing Machine) and a NTM (nondeterministic Turing Machine) are. You can read about them in the text §1.2 and 2.1.2 (first paragraph).

3. On the first day, I mentioned that **IP** = **PSPACE**. Each of these objects is a set of languages. Describe the structure of a proof that shows that **A** = **B**, where **A** and **B** are sets of languages. (The point of this problem is to remind you about how to do proofs with sets. The classes **IP** and **PSPACE** are just the motivation!)

4. What is the decision problem associated with the following language? (See final page for some notation information.) Please describe the problem in as natural non-mathy terms as possible.

$$\{\langle p, q, k\rangle : (\exists n : p, q \in \{0,1\}^n) \wedge (\exists S \subseteq [n] : (|S| \geq k) \wedge (\forall i \in S, p_i = q_i))\}. \tag{1}$$

5. The problem "k-Clique" takes as input an undirected graph and decides if there is a set of at least k vertices where there is an edge between each pair of vertices in the set. Write the language associated with this problem. (Please use set builder notation, and as much math notation as possible.)

6. (This problem is moved to Pset 2) * Prove that the following language is in **P**:

$$\mathsf{UNARYFACTORING} = \{\langle n_{\text{unary}}, l_{\text{unary}}, k_{\text{unary}}\rangle : \text{ there is a prime } j \in (l, k) \text{ that divides } n\}. \tag{2}$$

   In this case, n_{unary} means the number n represented in unary. So for example, 2 in base 10 is represented in unary as 11, and 5 in base 10 is represented in unary as 11111. Then $\langle 1111111111, 1111, 1111111\rangle \in \mathsf{UNARYFACTORING}$ because in base 10, this corresponds to the sequence $< 10, 4, 7 >$. Since 5 divides 10 and $5 \in (4, 7)$, this triplet satisfies the conditions, and so is in the language. On the other hand, $\langle 1111111111, 1111, 1111111\rangle \notin \mathsf{UNARYFACTORING}$ because there is no number between 4 and 7 that divides 9.

7. (Extra Practice Problem. Moved to Pset 2) Let $L_\triangle = \{G : G \text{ contains a triangle}\}$. Prove $L_\triangle$ is in **P**

Notation:

- $[n] = \{1, 2, 3, \ldots, n\}$
- $\{0,1\}^n$ is the set of n-bit binary strings.
- If $x \in \{0,1\}^n$, then for $i \in [n]$, x_i is the value of the ith bit of x.
- If $i, j \in \mathbb{Z}$ $(i, j]$ is the range of numbers from i to j including j but excluding i.