CS333 - Problem Set 9

1. Quantum Hype Chapter III: Frameworks for thinking about hype

   Please read the sections "Sources of Hype" and "Recognizing that Hype Matters" in Spinning the Genome: Why Science Hype Matters by Timothy Caulfield. Then also read Sections 3,4,5 of Could Hype be a Force for Good? by Tara Roberson.

   Please reflect on some or all of the following questions:

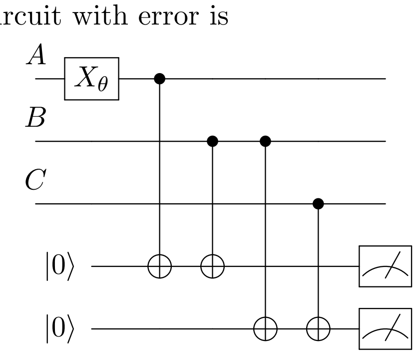
   - In what ways is hype and its effects similar in genomic research and quantum computing research? In what ways is it different? Please discuss in the context of some of the examples of hype that we have seen in previous "Chapters."
   - In what ways might hype around quantum computing cause harm (either to society or to the future of quantum computing research)? In what ways might hype around quantum computing cause benefit (either to society or to the future of quantum computing research)?
   - What do you think of Dr. Roberson's suggestion that hype is a potentially productive way to create dialogue between scientific communities and the public?
   - How do companies that are building quantum computers play into the hype landscape?

   We will discuss your responses, along with your thoughts on Quantum Hype I and II, in class on May 13.

2. In class, we showed how to create a 3-qubit quantum error correcting code that protected against an X error on any of the three qubits. In this problem, you will show that this code also protects against any error that is a rotation about the $\pm\hat{x}$ axis of the Bloch sphere. That is, an error of the form:

$$X_\theta = \begin{pmatrix} \cos(\theta) & i\sin(\theta) \\ i\sin(\theta) & \cos(\theta) \end{pmatrix}. \tag{1}$$

   (a) Suppose that an error X_θ occurred on the qubit A before running the error correction scheme. So the effective circuit with error is



(2)

If the input state to the circuit in Eq. (2) is $a|000\rangle_{ABC}+b|111\rangle_{ABC}$, what are the possible measurement outcomes of the final two qubits, and what state does the ABC system collapse to for each possible outcome? You can either analyze the circuit using the partial measurement framework, or using the equivalent projective measurement on ABC, $M = \{|000\rangle\langle 000| + |111\rangle\langle 111|, |001\rangle\langle 001| + |110\rangle\langle 110|, |010\rangle\langle 010| + |101\rangle\langle 101|, |100\rangle\langle 100| + |011\rangle\langle 011|\}$.

(b) Depending on the measurement outcome, what should you do to recover the state $a|000\rangle + b|111\rangle$ on system ABC?

(c) The calculation is similar if X_θ occurs on B or C. What are the possible measurement outcomes in each case, and how should you correct the error based on the measurement outcome? (You do not need to do any calculations, just state what happens and what you should do, given the results of part a/b, and the analysis we did in class.)

(d) Try to describe in general terms what is going on. For example, note that there are an infinite number of possible errors, since θ can be any real number between 0 and $\pi/2$. However, we can correct all of these errors by doing this single circuit with only a couple of possible correction operations, and without knowing θ. What role is the collapse playing in the error correction process?

3. (a) Explain why the circuit from the previous question does not detect (and hence can not correct) single qubit errors that are rotations of the $\pm\hat{z}$ axis of the Bloch sphere. That is, consider an error of the form:

$$Z_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \tag{3}$$

that acts where the X_θ acts in the previous question. If you repeat the analysis from the prior question, what outcome will you get when you measure? (You can use either the partial measurement or projective formalism.) Why can you not correct/detect this error?

(b) Show that if you encode the logical qubit $a|0\rangle + b|1\rangle$ in the physical encoded qubits $a|{+}{+}{+}\rangle_{ABC} + b|{-}{-}{-}\rangle_{ABC}$ (which you can do using a similar circuit as the one we've seen, but with H's thrown in) and then a Z_θ error occurs on qubit A, the projective measurement

$$\begin{aligned} M_H =\{&|{+}{+}{+}\rangle\langle{+}{+}{+}| + |{-}{-}{-}\rangle\langle{-}{-}{-}|, \\ &|{+}{+}{-}\rangle\langle{+}{+}{-}| + |{-}{-}{+}\rangle\langle{-}{-}{+}|, \\ &|{+}{-}{+}\rangle\langle{+}{-}{+}| + |{-}{+}{-}\rangle\langle{-}{+}{-}|, \\ &|{-}{+}{+}\rangle\langle{-}{+}{+}| + |{+}{-}{-}\rangle\langle{+}{-}{-}|\} \end{aligned} \tag{4}$$

on ABC can be used to detect and correct this error (The analysis for a Z-type error on the second or third qubits of the encoded state is similar - but you only need to analyze the first qubit case for this problem.)

Note that (up to a global phase, and using the identities $\cos(x) = (e^{ix} + e^{-ix})/2$ and $\sin(x) = (e^{ix} - e^{-ix})/(2i)$),

$$\begin{aligned} Z_\theta|{+}\rangle &= \cos(\theta/2)|{+}\rangle - i\sin(\theta/2)|{-}\rangle \\ Z_\theta|{-}\rangle &= - i\sin(\theta/2)|{+}\rangle + \cos(\theta/2)|{-}\rangle \end{aligned} \tag{5}$$

(c) The code in this problem can correct single Z-type errors, and the code from the previous problem corrects single X-type errors. In class, we discussed the 9-qubit Shor code, which corrects both single qubit X and Z errors (and in fact any single qubit error, since any single qubit error can be written as a sequence of X and Z errors). Looking at the logical encoding of the 9-qubit code:

$$\begin{aligned}|0_L\rangle &= (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) \\ |1_L\rangle &= (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)\end{aligned} \tag{6}$$

how does it combine aspects of both the X-type error correcting code, and the Z-type error correcting code?