CS333 - Problem Set 1

Due: Wed, Feb 21

Some of the problems require a basic knowledge of probability and complex numbers. Resources on both of these topics are on the ereserves page.

1. **[3 points each]** For each of the following numbers: plot in the complex plane, give it's complex conjugate, give it's absolute value, write in the form $a + bi$ (if not already in that form) and $re^{i\theta}$ (if not already in that form)

   (a) $2i - 2$
   (b) $e^{i\pi/4}$
   (c) $\frac{1}{i-1}$

2. **[6 points]** Consider a message m that consists of one bit, so $m \in \{0,1\}^1$, and a secret key $s \in \{0,1\}^1$. Using the XOR encryption from class, create a table that shows what $\overline{m}$ is for every possible message/key combination. Use this table to argue that Eve can not determine m from $\overline{m}$ if she knows nothing about the value of m or s. (You may also use Baye's rule, if you know it, but this is not necessary.)

3. **[6 points]** Suppose Alice sends Bob a photon that is either vertically, horizontally, right-diagonally, or left-diagonally polarized. Suppose Bob puts a vertically polarized filter in front of a single photon detector. If Bob's detector's light turns on, what does he know about the polarization of the photon Alice sent? If the detector's light does not turn on, what does he know about the polarization of the photon Alice sent?

4. **[6 points]** Suppose Alice sends a vertically polarized photon through a diagonally polarized filter, followed by a vertically polarized filter. What is the probability that a photon will exit the second filter, and what will its polarization be?

5. **[6 points]** Suppose Eve uses the following strategy: with probability p, she measures using a vertically polarized filter, and then passes on a vertically or horizontally polarized photon based on the outcome of her measurement (this is the option "0" from class), and with probability $1 - p$, she does nothing and just lets the photon pass through to Bob (option "2" from class.) Given this strategy, what is the probability that a bit of b' will differ from the corresponding bit in d'? Assume Alice's and Bob's strategy is the same as in class.

6. **[6 points]** In your own words, describe what it is about quantum states that makes it possible for Alice and Bob to share a secret key over a public channel (a public channel is a channel where eavesdropping is possible).

7. **[6 points]** Read The Space-Based Quantum Cryptography Race and explain (1) why scientists are trying create a quantum cryptography satellite and (2) why is it important that the error rate is below some threshold?

8. How long did you spend on this homework?