Rest of Class:

- Period Finding Alg

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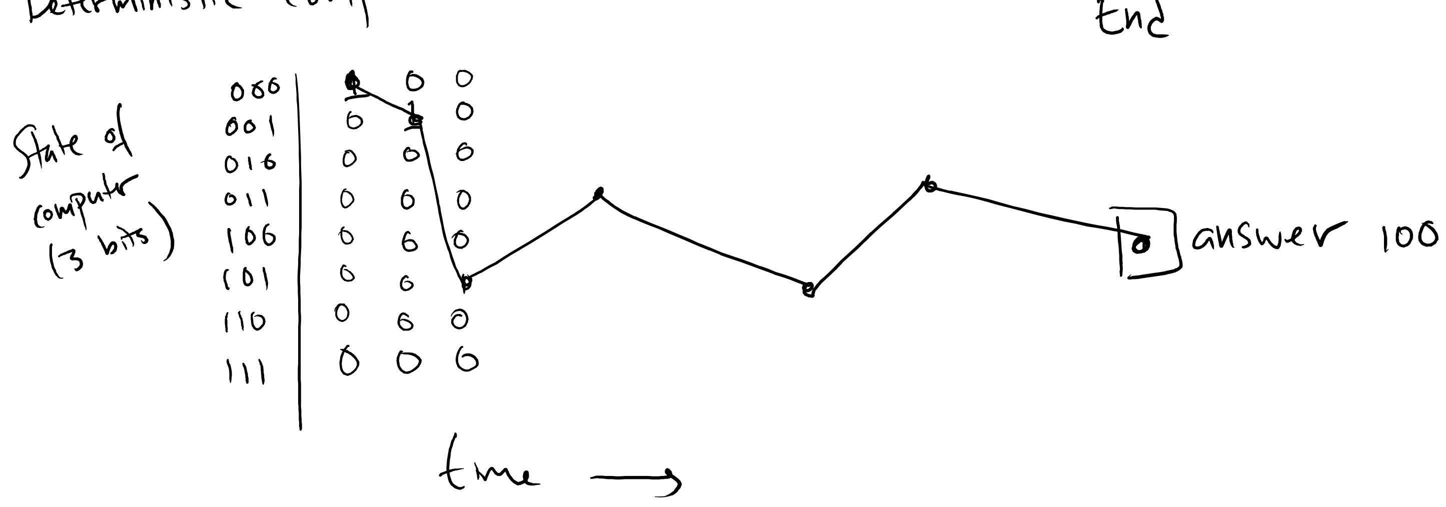
Answer Questions

Goals

Describe computation in terms of Feynman path Integrals

Computation as Path

Deterministic computation:



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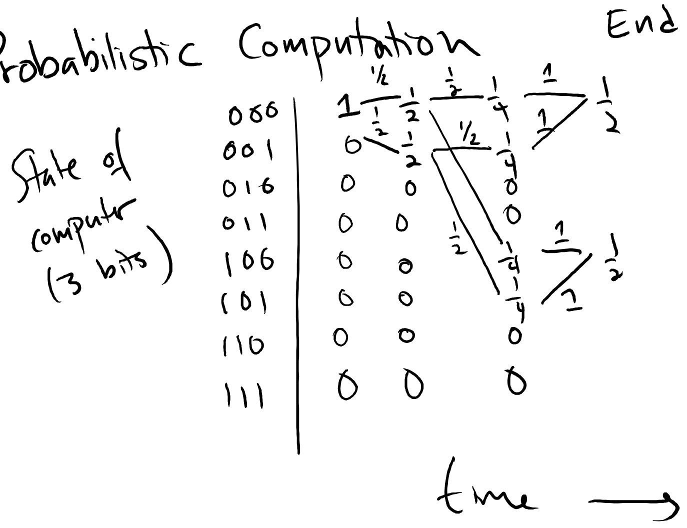
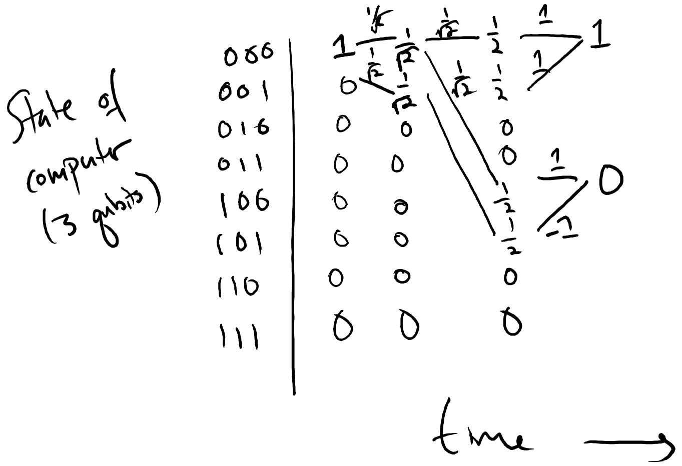
Probabilistic Computation

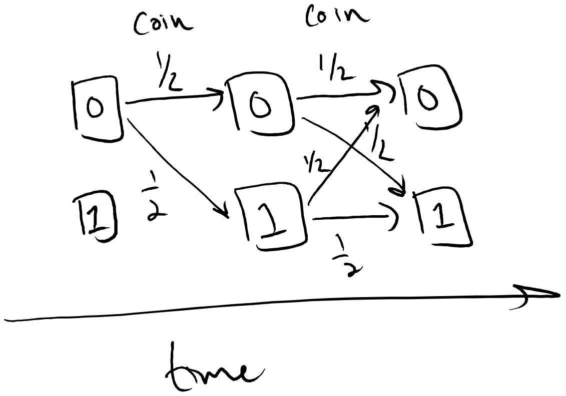


Quantum Computation



Why does quantum algorithm do better?

Let's compare $-\boxed{H}-\boxed{H}-$ to 2 coin flips $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$



Coin action

$$\boxed{0} \rightarrow \frac{1}{2}\boxed{0} + \frac{1}{2}\boxed{1}$$

$$\boxed{1} \rightarrow \frac{1}{2}\boxed{0} + \frac{1}{2}\boxed{1}$$

1st flip

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2nd flip

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\left[\frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = \left[\frac{1}{4}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$$

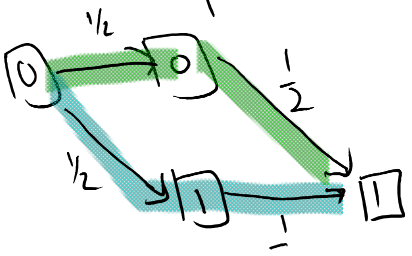
Combine paths that end at same point:

$$\frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

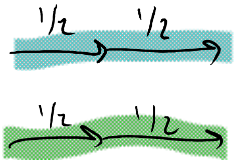
One way to determine final probability of $\overset{\text{Start}}{|0\rangle} \longrightarrow \overset{\text{end}}{|1\rangle}$

- Look at all paths from start to end

2 paths



- Probability of traversing a path is product of probabilities on each edge

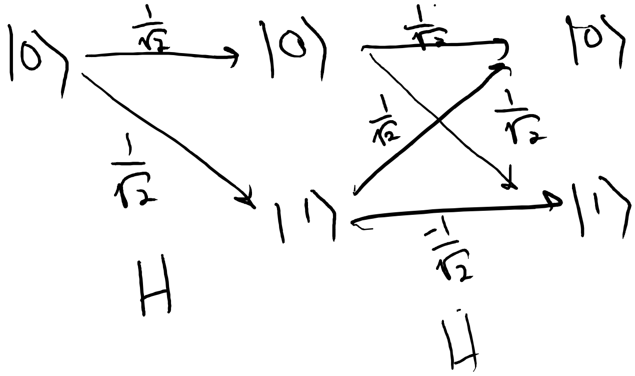


Prob

$1/4 = 1/2 \cdot 1/2$

$1/4 = 1/2 \cdot 1/2$

Now quantum version:



H's action:

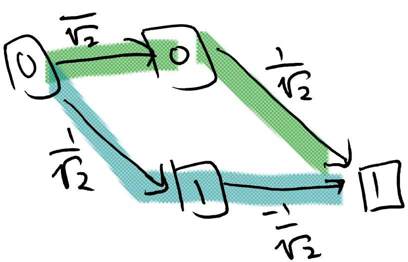
$|0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

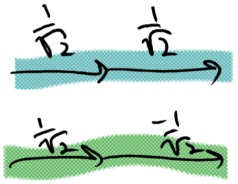
One way to determine final probability of $\overset{\text{Start}}{\boxed{0}} \longrightarrow \overset{\text{end}}{\boxed{1}}$

- Look at all paths from start to end

2 paths



- Amplitude of traversing a path is product of on each edge

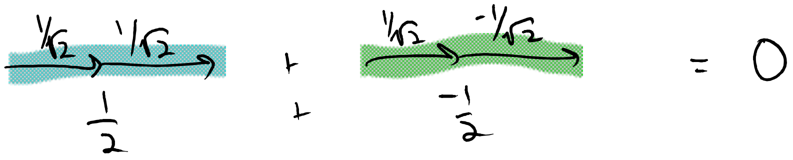


Amplitude

$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$

$\frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{2}} = \frac{-1}{2}$

- Total amplitude is sum of amplitudes of each path:



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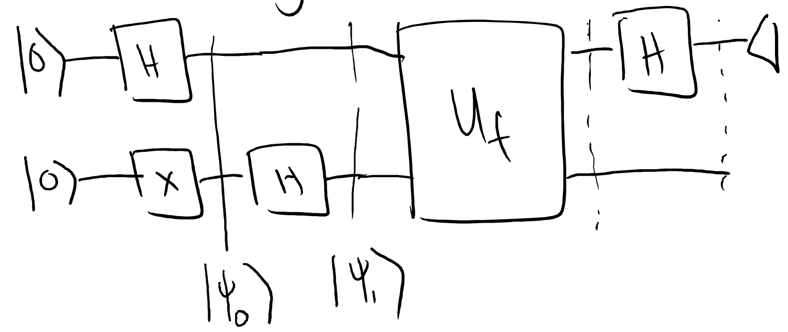
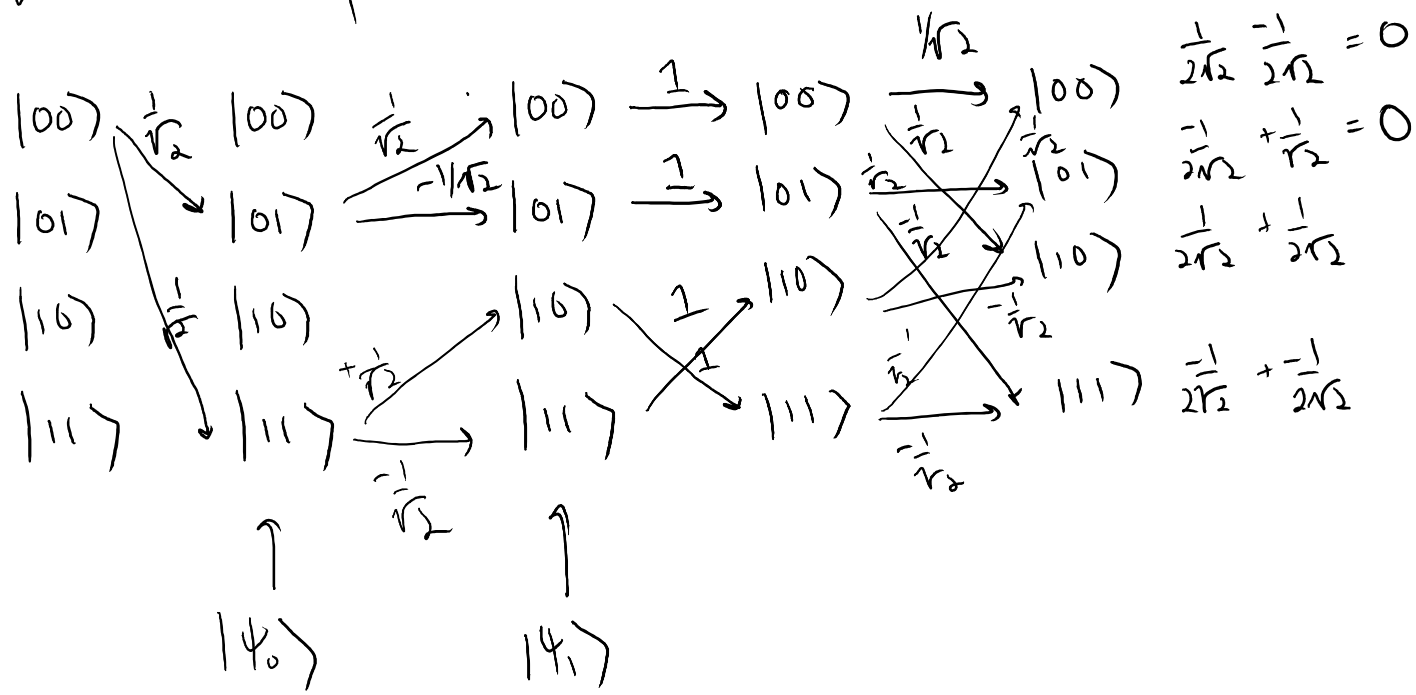
Deutsch Algorithm



What are paths if $f(x)=x$? Other values of f?



Take-away:

Quantum computers don't get power from exploring many paths - probabilistic computers can do that too. The power comes from having paths interfer and cancel out.

PSET: Show $\nexists$ a U s.t.

perfectly distinguishes