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Q: Why won't the same strategy work for Quantum

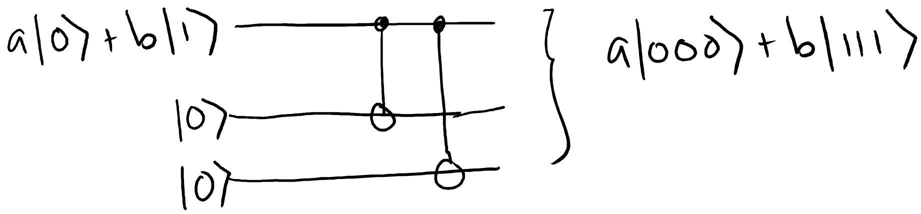
A) MAJ not reversible

B) We can have more varied errors in quantum case

C) No FANOUT because no cloning

D) All of the above

Instead, use this encoding:



Encode $a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$

<u>Good</u>:
- Don't need to measure input to encode.
- If $a=0$ or $b=0$ $\rightarrow$ same as classical.

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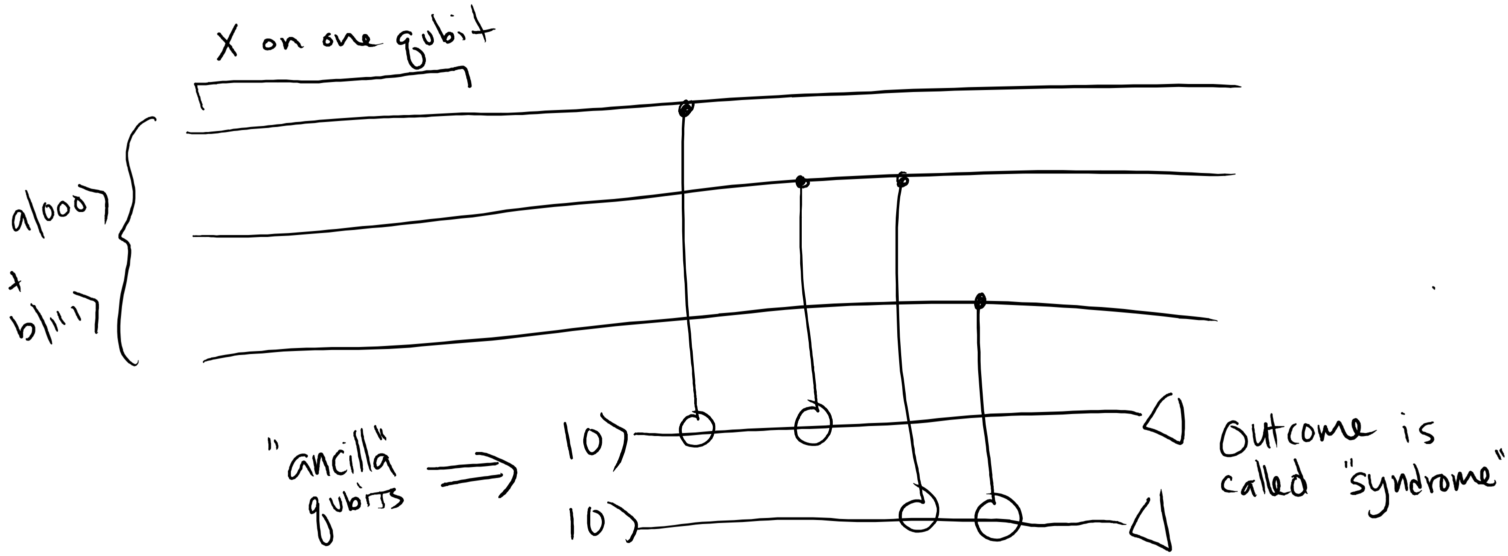

1. What measurement outcomes do you get with which probability?
2. What does the rest of the state collapse to?
3. How do you fix error based on outcome
4. What happens if Z error on a qubit?

	1.	2.	3.
• If $\boxed{X}$ on 1st,	Outcome $\|10\rangle$ →	$a\|100\rangle + b\|111\rangle$ →	X on 1
• If $\boxed{X}$ on 2nd,	outcome $\|11\rangle$ →	$a\|010\rangle + b\|101\rangle$ →	X on 2
• If $\boxed{X}$ on 3rd	outcome $\|01\rangle$ →	$a\|001\rangle + b\|001\rangle$ →	X on 3
• If no X,	outcome $\|00\rangle$ →	$a\|000\rangle + b\|111\rangle$ →	nothing

4. Get outcome $|00\rangle \Rightarrow$ no detection

Don't need to know a, b

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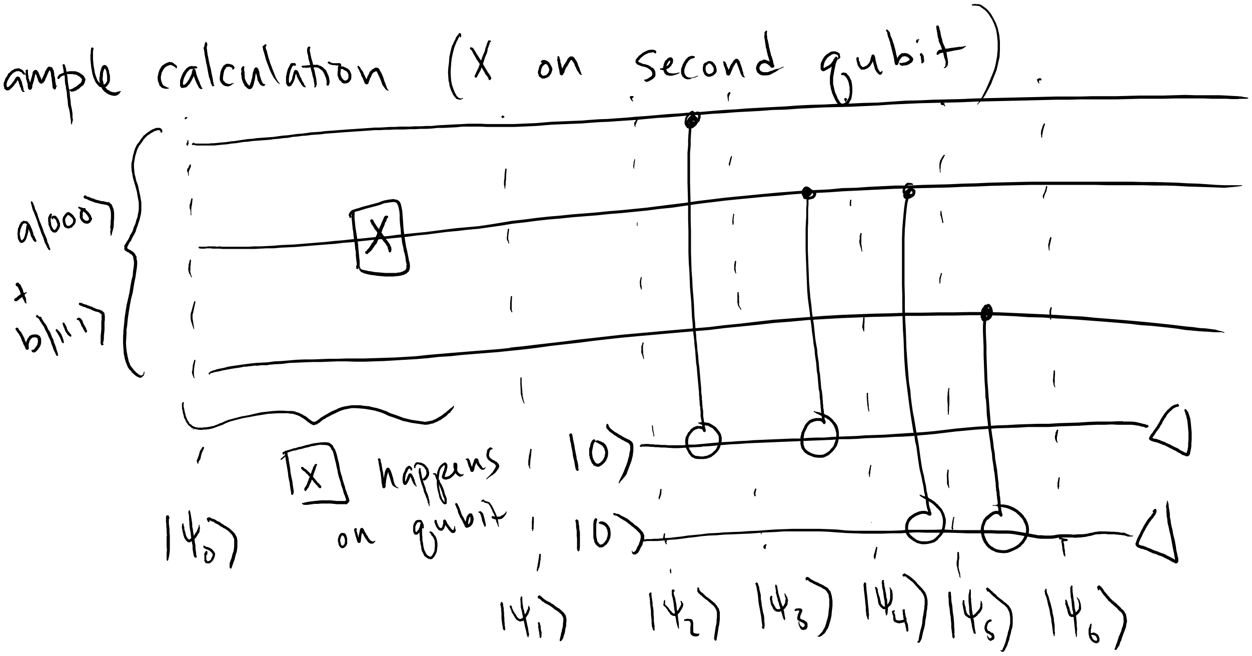
Sample calculation (X on second qubit)



$$|\psi_1\rangle = X_2\left(a|000\rangle + b|111\rangle\right) = a|010\rangle + b|101\rangle$$

$$|\psi_2\rangle = a|010\rangle|00\rangle + b|101\rangle|00\rangle$$

$$|\psi_3\rangle = a|010\rangle|00\rangle + b|101\rangle|10\rangle$$

$$|\psi_4\rangle = a|010\rangle|10\rangle + b|101\rangle|10\rangle$$

$$|\psi_5\rangle = a|010\rangle|11\rangle + b|101\rangle|10\rangle$$

$$|\psi_6\rangle = a|010\rangle|11\rangle + b|101\rangle|11\rangle$$

$$= \left(a|010\rangle + b|101\rangle\right)|11\rangle$$

Apply $X \rightarrow a|000\rangle + b|111\rangle$

New mathematical description of partial measurement

(Mathematically equivalent to appending ancillas, interacting, measure ancillas)

Old: $M = \{|\phi_i\rangle\}$ Outcome i with prob $|\langle\psi|\phi_i\rangle|^2$

orthonormal basis

$|\psi\rangle \to |\phi_i\rangle$

New: $M = \{P_i\}$

orthonormal states

$$P_i = \sum_{k \in S_i} |\phi_k\rangle\langle\phi_k|$$ (P_i are "projectors")

$$\sum P_i = \mathbb{I}$$ ($|\phi_k\rangle\langle\phi_k|$ appears in exactly $\underline{1}$ projector)

$$P_i P_j = \begin{cases} 0 & i \neq j \\ P_i & i = j \end{cases}$$

Outcome $i \rightarrow Pr(i) = \langle\psi|P_i|\psi\rangle$

$$|\psi\rangle \rightarrow \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$$

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Measurement for Error Correction of X:

$$M = \{ \underbrace{|000\rangle\langle 000| + |111\rangle\langle 111|}_{P_0},\ \underbrace{|100\rangle\langle 100| + |011\rangle\langle 011|}_{P_1},\ \underbrace{|010\rangle\langle 010| + |101\rangle\langle 101|}_{P_2},\ \underbrace{|001\rangle\langle 001| + |110\rangle\langle 110|}_{P_3} \}$$

$P_0 \leftrightarrow$ Outcome $|00\rangle$ on ancilla

$P_1 \leftrightarrow$ " $|10\rangle$

$P_2 \leftrightarrow$ " $|11\rangle$

$P_3 \leftrightarrow$ " $|01\rangle$

- Only 4 outcomes
- 8-Dim space

} does not fully collapse

Q: If measure $a|000\rangle + b|011\rangle + c|100\rangle$ with M, which outcomes are possible?

A) P_0, P_1 $\Leftarrow$ all others $P_i\left(a|000\rangle + b|011\rangle + c|100\rangle\right) = 0$

B) P_1, P_2

C) P_0, P_2

D) P_1, P_3

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Q: If get outcome P_1 when measure $a|000\rangle + b|011\rangle + c|100\rangle$, what does state collapse to?

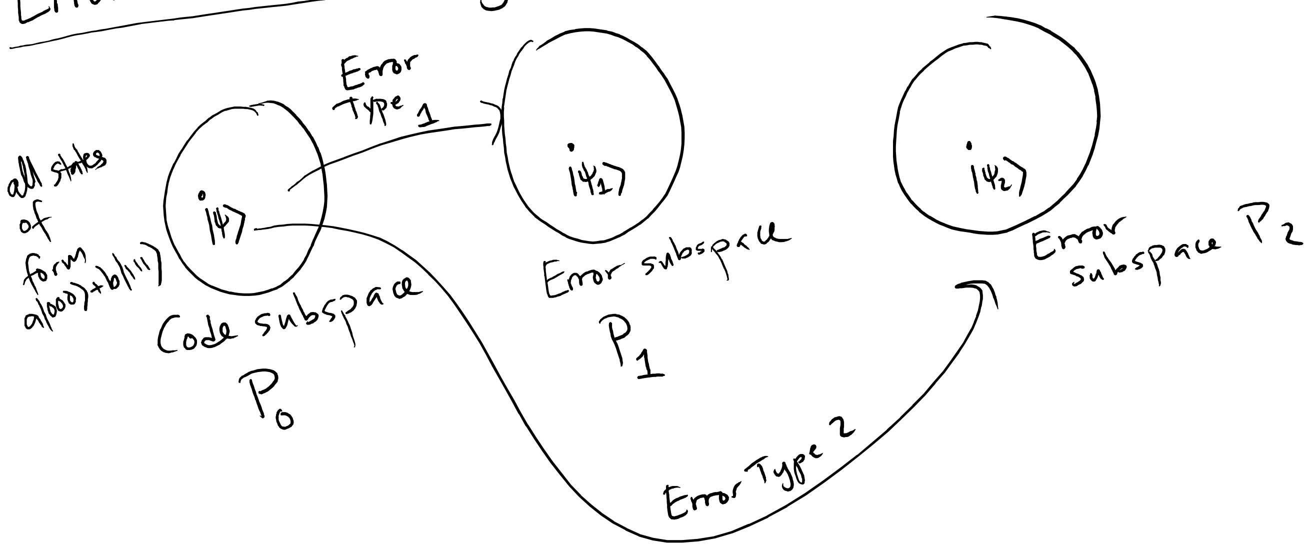
A) $b|011\rangle + c|100\rangle$

B) $\left(b|011\rangle + c|100\rangle\right)\frac{1}{\sqrt{|b|^2+|c|^2}} \Leftarrow \frac{P_1|\psi\rangle}{\sqrt{\langle\psi|P_1|\psi\rangle}}$

c) $b|011\rangle$

d) $c|100$

Error Correction Big Idea



Measurement doesn't cause full collapse, just tells you type of error. Doesn't tell you about a, b.