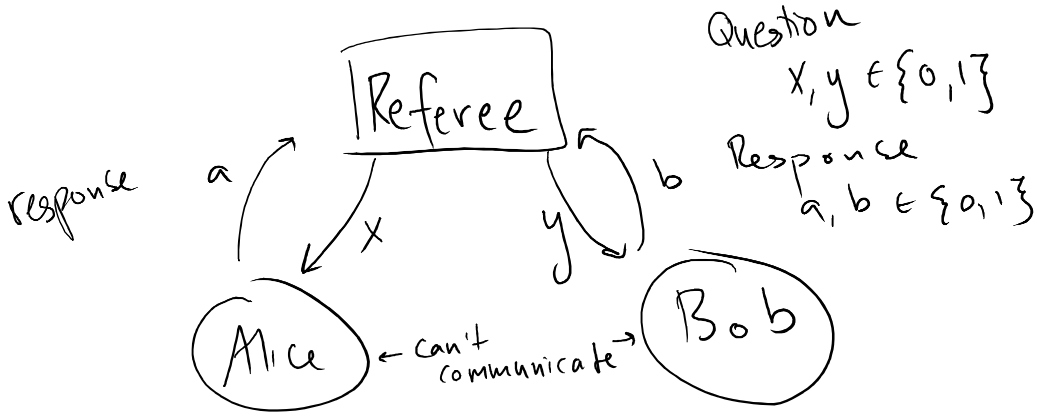
S. KIMMEL

More Qubits!

1 qubit at a time → better crypto

2 " " → better game playing



Alice & Bob win if $x \wedge y = a \oplus b$

x	y	$x \wedge y$	$a \oplus b$
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Q: Figure out the best strategy for Alice and Bob, averaged over choice of x, y, chosen uniformly at random

A: Best strategy, always choose $a=0$ $b=0$. Will win 75% of time

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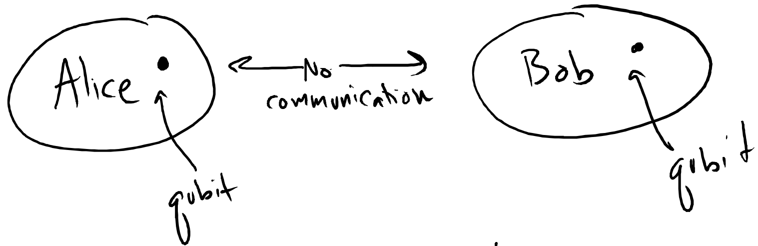
Now:



Can they do better? ... Yes!

S. KIMMEL

Need math to describe $\boxed{2 \text{ qubits}}$:

$\otimes$ = "tensor product":

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae & af & be & bf \\ ag & ah & bg & bh \\ ce & cf & de & df \\ cg & ch & dg & dh \end{pmatrix}$$

Qubit A ↓ Qubit B ↓

$$|\psi_1\rangle_A = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \qquad |\psi_2\rangle_B = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$$

Qubit A and B together ↓

$$|\psi\rangle = |\psi_1\rangle_A \otimes |\psi_2\rangle_B = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix} = a_0 b_0 |00\rangle_{AB} + a_0 b_1 |01\rangle_{AB} + a_1 b_0 |10\rangle_{AB} + a_1 b_1 |11\rangle_{AB}$$

More: leave out $\otimes$, OK!

$$|0\rangle_A |0\rangle_B = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|01\rangle_{AB} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

just combine, OK!

$$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$$

orthonormal basis

"standard basis"

S. KIMMEL

Properties of Tensor Product

- Q: What is $|1\rangle \otimes \left(\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle\right)$?

A: $\frac{1}{\sqrt{3}}|01\rangle + \sqrt{\frac{2}{3}}|11\rangle$

B: $\frac{1}{\sqrt{3}}|10\rangle + \sqrt{\frac{2}{3}}|11\rangle$

C: $\frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$

D: $\frac{1}{\sqrt{3}}|10\rangle + \sqrt{\frac{2}{3}}|01\rangle$

Distributive

- Q: If $|\psi\rangle = |\psi_1\rangle_A |\psi_2\rangle_B$, what is $\langle\psi|$?

A: $\langle\psi_1|_A \otimes \langle\psi_2|_B = \langle\psi_1|_A\langle\psi_2|_B$

B: $\langle\psi_2|_B \otimes \langle\psi_1|_A = \langle\psi_2|_B\langle\psi_1|_A$

conj. transpose of tensor product is tensor product of conj. transposes

- Q: If $|\psi\rangle = |\psi_1\rangle_A |\psi_2\rangle_B$ is a 2-qubit state what is $\langle\psi|\psi\rangle$?

A: $|\langle\psi_1|\psi_2\rangle|^2$

B: $\langle\psi_1|\psi_1\rangle \cdot \langle\psi_2|\psi_2\rangle$

Inner product of tensor product is product of inner products

$$\left(\langle\phi_1|_A\langle\phi_2|_B\right)\left(|\psi_1\rangle_A|\psi_2\rangle_B\right) = \langle\phi_1|\psi_1\rangle\cdot\langle\phi_2|\psi_2\rangle$$