Knapsack Problem ~~$\mathbb{N}$~~ $\mathbb{R}$

Input:

- n items
  - $v_i \in$ ~~$\mathbb{N}$~~ $\mathbb{R}$ is value of i^{th} item
  - $w_i \in$ ~~$\mathbb{N}$~~ $\mathbb{R}$ is weight of i^{th} item
- Capacity W max weight $\in$ ~~$\mathbb{N}$~~ $\mathbb{R}$ (ok)

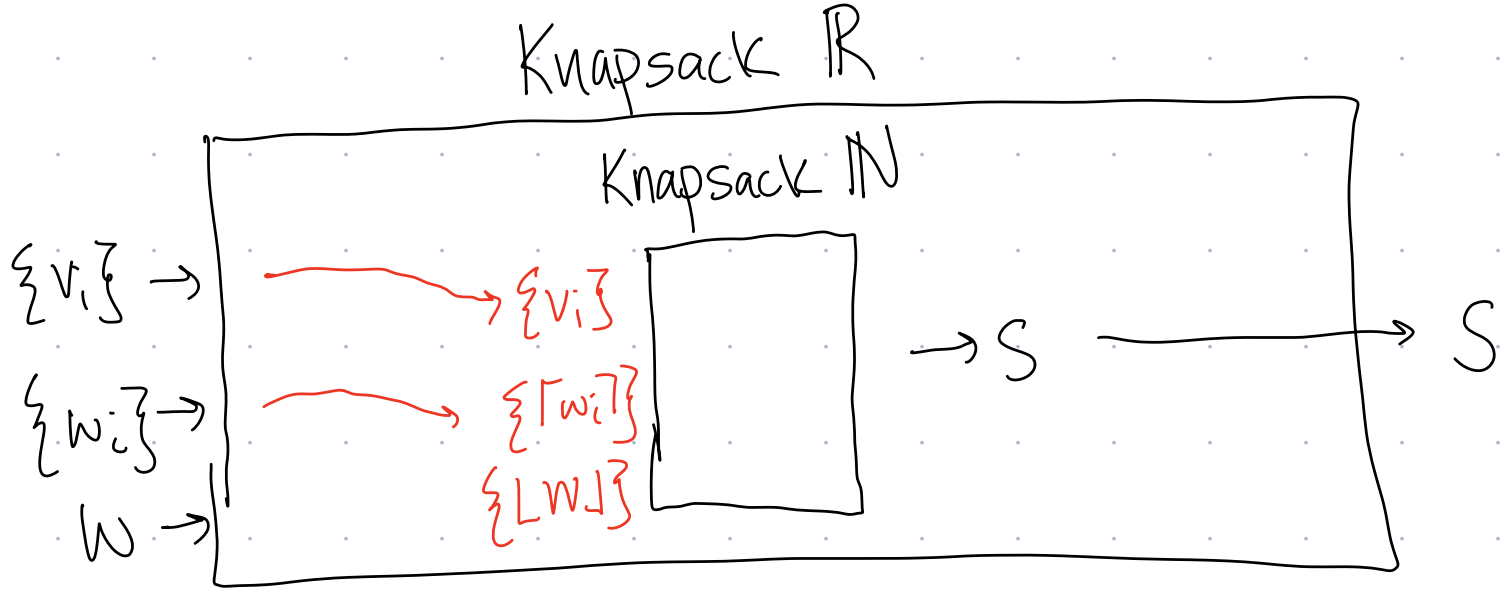
Output:

$S \subseteq [n]$ $\quad [n] = \{1, 2, \ldots n\}$

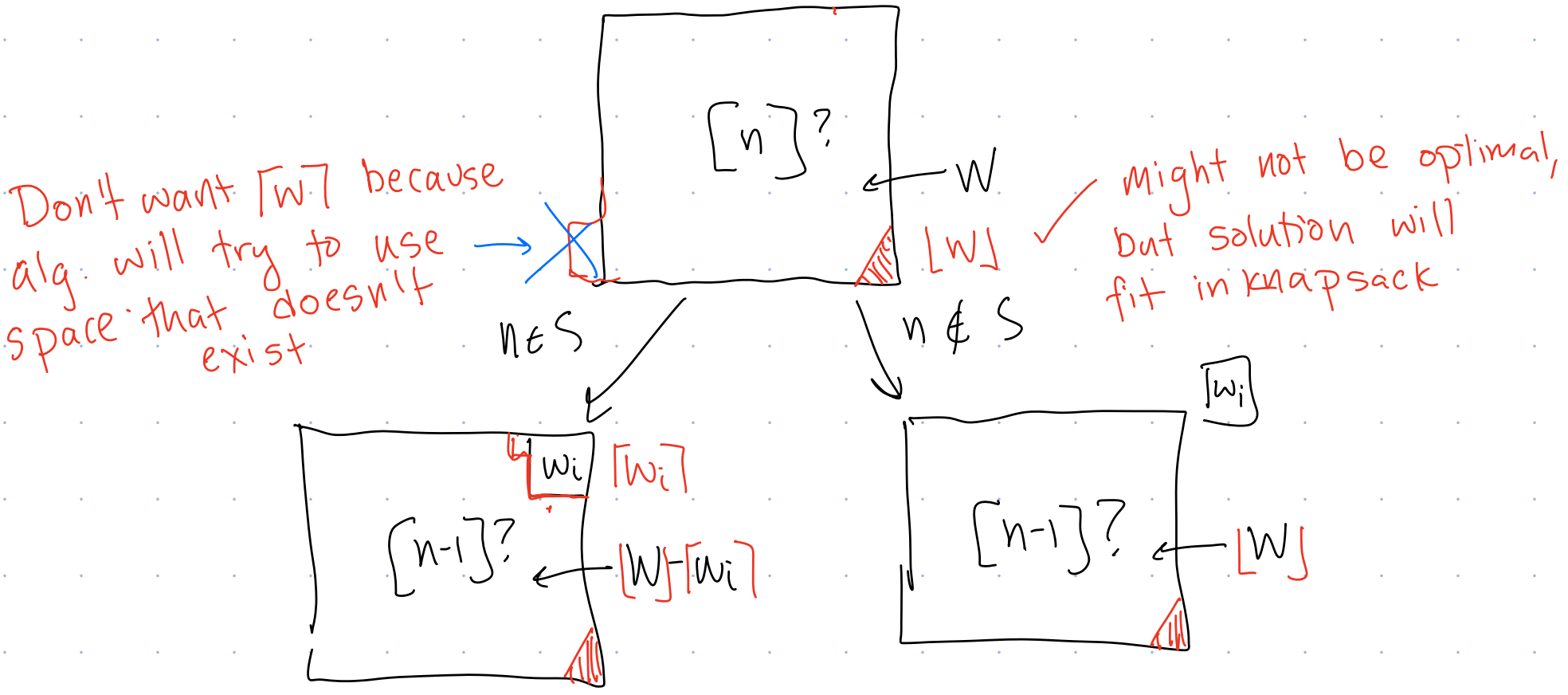
s.t. $V(S) = \sum_{i \in S} v_i$ is close to maximized

$W(S) = \sum_{i \in S} w_i \leq W$ ✓

Reduce Knapsack $\mathbb{R}$ to Knapsack $\mathbb{N}$



How can you improve your approximate alg? (At cost of time)


Don't want ⌈w⌉ because alg. will try to use space that doesn't exist
[n]?
W
⌊W⌋
Might not be optimal, but solution will fit in knapsack
n ∈ S
n ∉ S
wi
⌈wi⌉
[n-1]?
W-wi
wi
[n-1]?
W

$$A[n, W] = \max\{ A[n-1, W],\ A[n-1, W-w_n] + v_n \}$$

ex: $W=10,\ w_5=2$:

$$A[5,10] = \max\{ A[4,10],\ A[4, 10-2] + v_n \}$$

$$A[4, 8]$$

ex: $W=10 \quad w_5=1.7$

$$A[5,10] = \max\{ A[4,10],\ A[4, 10-1.7] + v_n \}$$

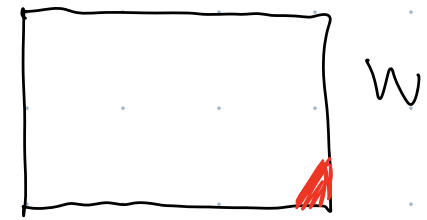
$$A[4, 8.3] \ !?$$

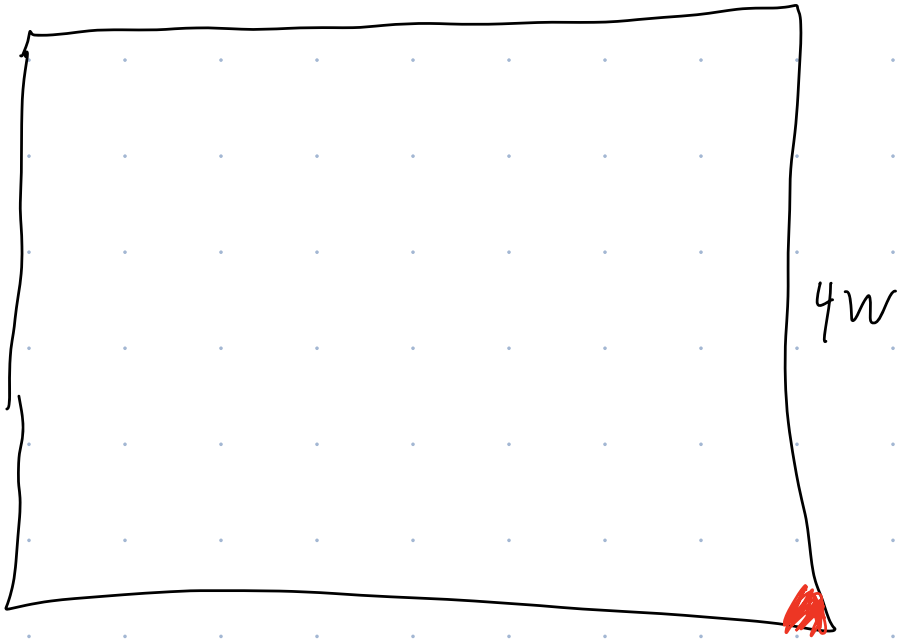
A

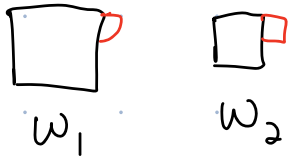
✡ when $W, w_i \to \mathbb{R}$, can't store capacity of knapsack in column of A ∴

✡ When $v_i \to \mathbb{R}$ OK! A just stores $\mathbb{R}$ instead of $\mathbb{Z}$

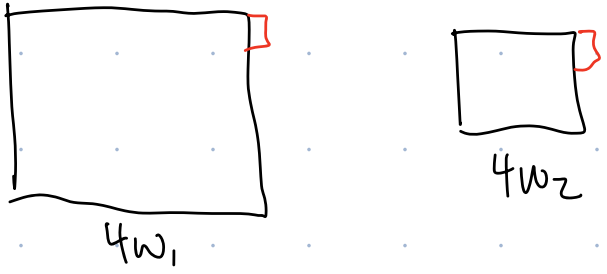
To improve approximation, scale up before rounding:







x4



Relative effect of rounding is decreased!

... but runtime increases by a factor of 4.