CS302 - Problem Set 11

1. Recall the following definitions:

   - k-INDSET: Given an undirected, unweighted graph $G = (V, E)$, and a number k, output YES if there is a set $V' \subseteq V$ such that $|V'| \geq k$, and for all $v, u \in V'$, there is no edge $\{u, v\} \in E$. (We call such a set a k-independent set.) Otherwise output NO.
   - k-CLIQUE: Given an undirected, unweighted graph $G = (V, E)$, and a number k, output YES if there is a set $V' \subseteq V$ such that $|V'| \geq k$, and for all $v, u \in V'$, there is an edge $\{u, v\} \in E$. (We call such a set a k-clique.) Otherwise output NO.

   In the following, you can use the results you already proved, that k-INDSET and k-CLIQUE are in NP.

   (a) Prove that there is a polynomial reduction from k-INDSET to k-CLIQUE.

   (b) Prove that there is a polynomial reduction from 3-SAT to m-INDSET, where m is the number of clauses in the 3-SAT.

   (c) Use the previous two parts to prove that k-INDSET and k-CLIQUE are NP-Complete.

   Hints

   (a) Look up the *complement* of a graph. It will be helpful.

   (b) Try to come up with a simple graph gadget for each clause (it should have one vertex for each variable in the clause). Then figure out how to connect the gadgets.

   (c) This is a short proof!

2. **Challenge:** Consider the problem of outputting the *second* largest value of an array A. A is an unsorted array of unique integers of length n where n is a power of 2.

   (a) Describe an algorithm that uses exactly $n + \log_2 n - 2$ comparison operations, where a comparison is an operation that tests whether one element is less than another element.

   (b) Justify why your algorithm is correct. (You can do this as a proof, or as a slightly less formal explanation.)