12s: NP Complete Problems

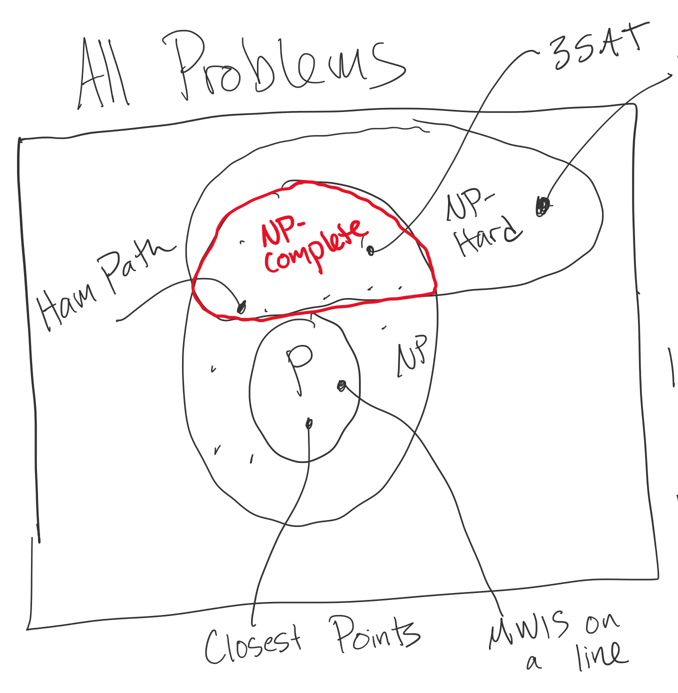
Friday, April 30, 2021
11:20 AM

Goals:

- Understand meaning of NP-Hard, NP-Complete
- Start describing reduction of 3SAT to HamPath

Announcements:

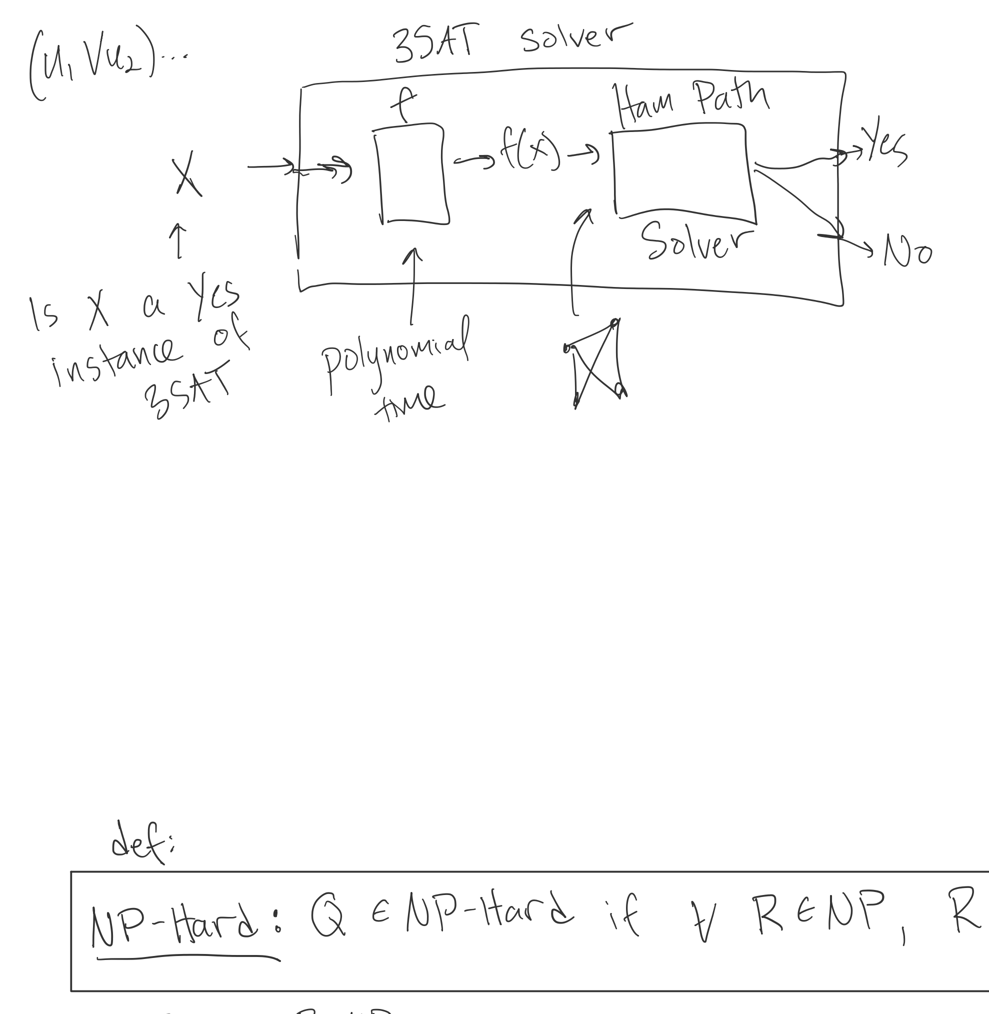
- Go/middhacks2022
- Great exit tickets on Friday- see pinned discussion for my responses
- Don't forget about quiz, midterm revisions! Don't leave for the last minute!



$3SAT \le_p$ Ham Path

HamPath $\le_p 3SAT$

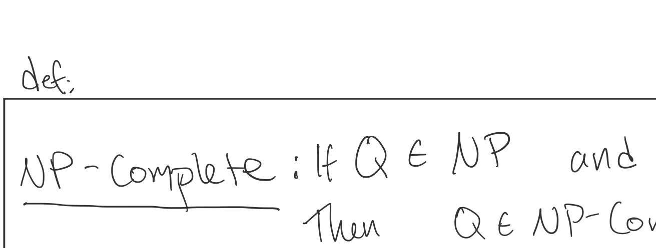
In fact, if find a fast alg for 3SAT or HamPath, you could use it to solve every other problem in NP.



def:

NP-Hard: $Q \in$ NP-Hard if $\forall R \in NP$, $R \le_p Q$

For all $R \in NP$



def:

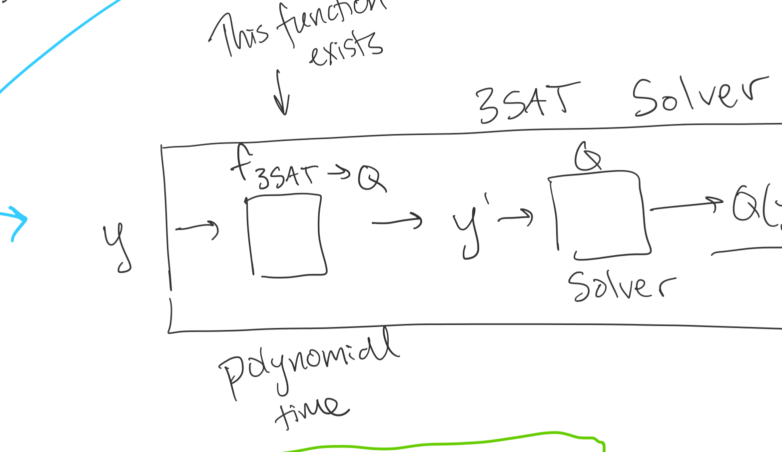
NP-Complete: If $Q \in NP$ and $Q \in$ NP-Hard
Then $Q \in$ NP-Complete

How?

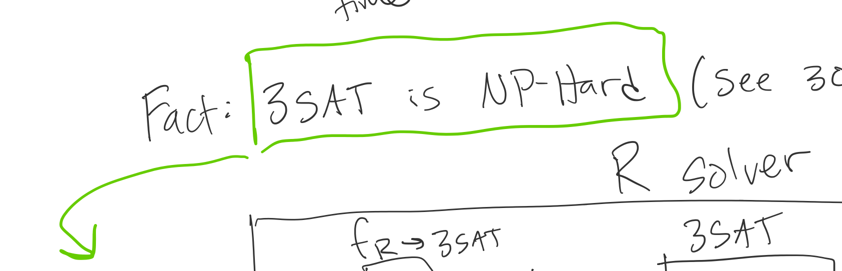
Show $3\text{-}SAT \le_p Q$

Theorem: • If $3SAT \le_p Q$ then $Q \in$ NP-Hard:

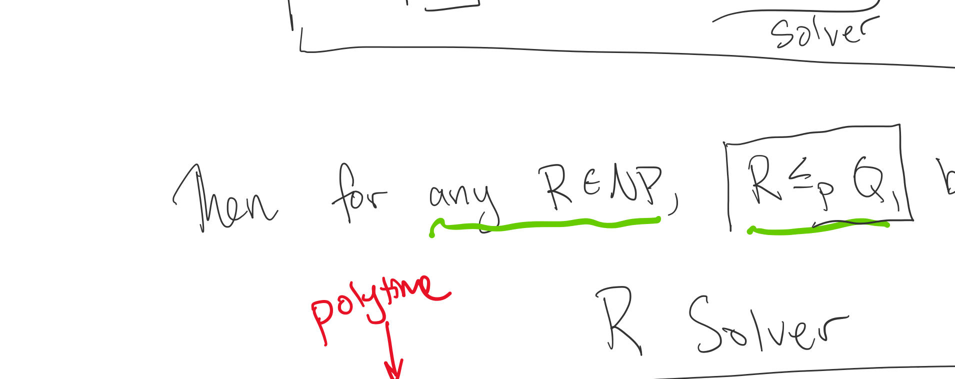
Pf: •



Fact: 3SAT is NP-Hard (See 301). So $\forall R \in NP$



Then for any $R \in NP$, $R \le_p Q$, because



So Q is NP-Hard.

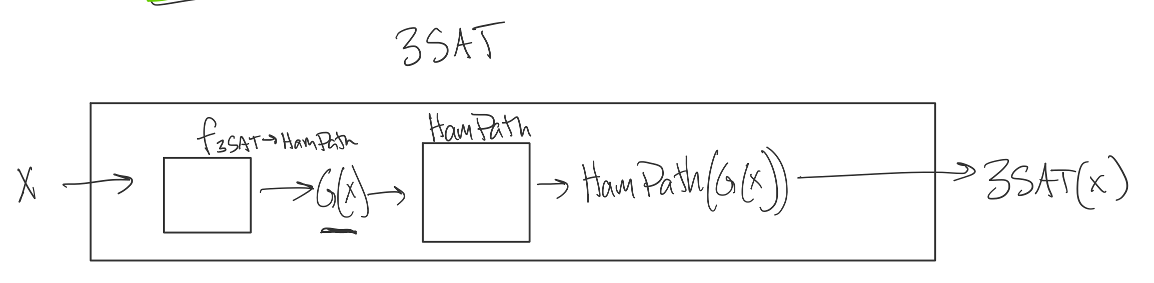
Group Work

1. Fill in box to finish proof.
2. Brainstorm ideas for $3SAT \le_p$ HamPath

Theorem: Hamitonian Path is NP-Complete

Pf • HamPath $\in NP$ ✓
• Ham Path $\in$ NP-Hard

↳ $3SAT \le_p$ HamPath



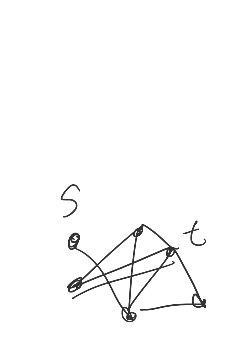
Need to prove this reduction exists

1. Describe $f_{3SAT \to HamPath}$
2. Analyze runtime of $f_{3SAT \to HamPath}$
3. Prove conversion is correct
   a) If HamPath(G(x)) = Yes → 3SAT(x) = Yes
   b) If 3SAT(x) = Yes → HamPath(G(x)) = Yes

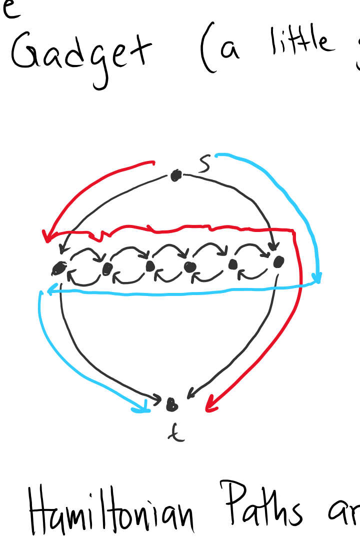
1. Describe $f_{3SAT \to HamPath}$

$(u_1 \vee \neg u_2 \vee u_6) \wedge (\dots) \to$

X



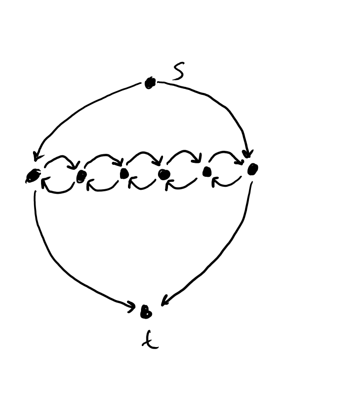
Idea: Use
Graph Gadget (a little graph that implements some functionality)

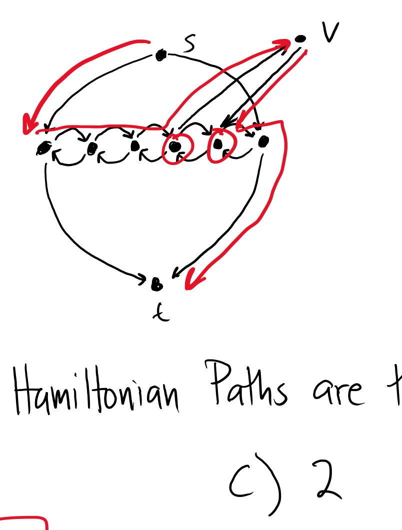


How many Hamiltonian Paths are there from s to t?

A) 0
B) 1
C) 2
D) >2

2 paths

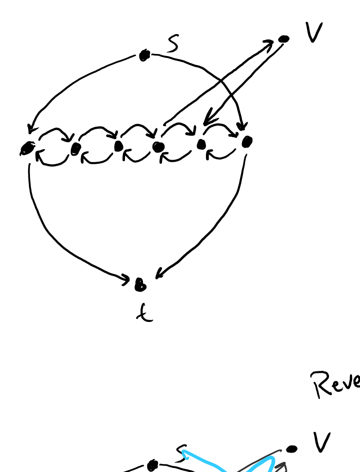




How many Hamiltonian Paths are there from s to t?

A) 0
B) 1
C) 2
D) >2

1 path



Reverse direction of edges to v.