

- Goals:
- Practice designing a DP algorithm
- Announcements:
- Prof. Linderman Today! 4:30 Hillcrest 103
 - Friday - discuss Search and Dr. Noble's research
 - Monday @7 on Zoom: Tapia Panel

Recurrence Relation for $S_{i,r}$

$$S_{i,r} = \begin{cases} S_{i-1,r} & \text{if } i \notin S_{i,r} \quad \leftarrow \\ \boxed{S_{i-1,r-c_i} \cup \{i\}} & \text{if } i \in S_{i,r} \quad \leftarrow \end{cases}$$

$\forall r, \underline{S_{0,r}} = \emptyset$ Base Case

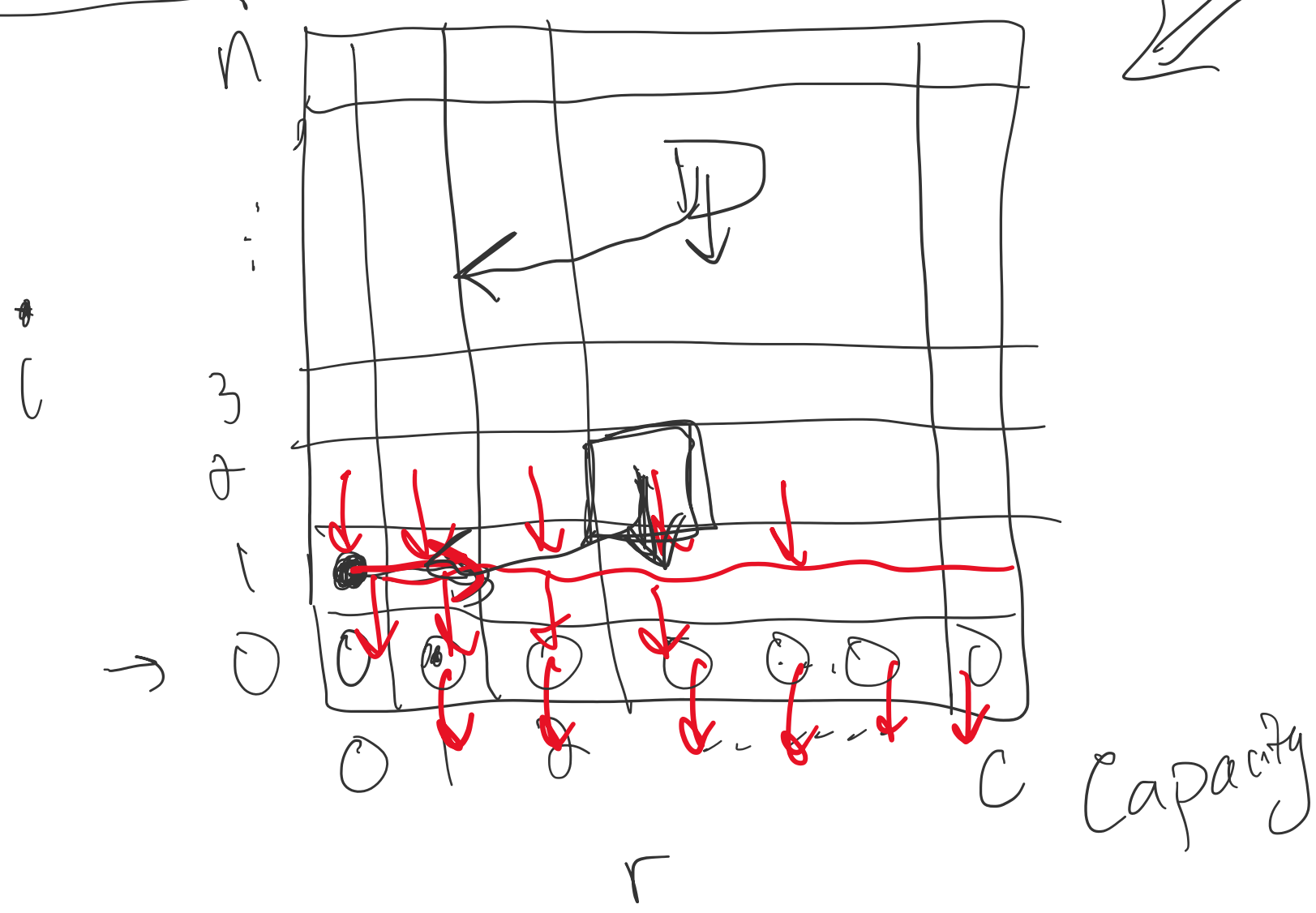
Recurrence Relation for Objective Function

$$\underline{V(S_{i,r})} = \max \left\{ \underline{V(S_{i-1,r})}, \underline{V(S_{i-1,r-c_i}) + v_i} \right\}$$

$\forall r, V(S_{0,r}) = 0$

Only an option
if $r \geq c_i$

$A \Rightarrow A[2,1] = \underline{V(S_{2,1})}$



Pseudocode

Input: $n \times 1$ arrays V (values)
and c (costs)

Output: Optimal set to put in knapsack

Initialize A as $(n+1) \times (C+1)$ array

// Base Case

For $r = 0$ to C
| $A[0,r] = 0$

For $i = 1$ to n :

| For $r = 0$ to C :

| | if $r \geq c_i$:

| | $\Rightarrow A[i,r] = \max \{ A[i-1,r], A[i-1,r-c_i] + v_i \}$

| | else:

| | $A[i,r] = A[i-1,r]$

Test

Ex: $C = 5$

Item	Value	Cost
1	6	2
2	5	1
3	8	3
4	7	4

Q: What is the runtime of DP knapsack algorithm?

- A) $O(n)$ B) $O(C)$ c) $O(n \cdot C)$ D) $O(n^2)$