3: Scheduling (Greedy)

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8:54 PM

In Groups

- Come up with at least 2 reasonable f-functions to minimize A.
  (use $+, -, \div,$ or $*$ of w_i, t_i)
- Test on

job	time	weight
1	5	2
2	3	1

(To be consistent, will order large f-value jobs first)

or otherwise try to rule out

$f_1 = w_i/t_i$ $\quad f_2 = w_i^{1/t_i}$ $\quad f_3 = w - t$

job	time	weight	f_1	f_3
1	5	2	2/5	−3
2	3	1	1/3	−2
			(1,2)	(2,1)

order	A
1 2	18 ✓
2 1	19

Goals: (w/t)

1. Prove our f function is optimal
2. Describe general approach for greedy algorithm proofs

Thm: Greedy alg with $f = w/t$ is optimal for obj. fun. $A(\sigma) = \sum_{i=1}^{n} w_i C_i(\sigma)$.

	job	time	weight	f	
(2)	~~1~~	5	2	2/5	next
(3)	~~2~~	3	1	1/3	smallest
(1)	3	9	4	4/9	largest

Pf: Exchange argument (contradiction)

Assume w_i/t_i are distinct $\forall\ i \in \{1, 2, \dots, n\}$

WLOG, relable so $\boxed{w_1/t_1 > w_2/t_2 > w_3/t_3 > \dots\ w_n/t_n}$

So let $\sigma = (1, 2, 3, \dots, n)$. This is the greedy ordering. For contradiction, assume σ is not optimal. Let σ^* be the optimal ordering. $\sigma \neq \sigma^*$

Then $\exists$ jobs j, k that are not in numeric order in σ^*

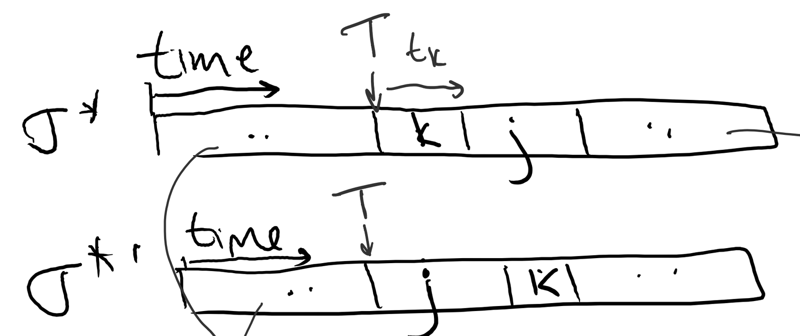
$\sigma^* = (\dots\dots, k, j \dots\dots)$ $\quad \boxed{j < k}$

Let $\sigma^{*\prime}$ be the same as σ^* but with j, k exchanged:

$\sigma^{*\prime} = (\dots\dots, j, k, \dots\dots)$ (same) (same)

$$A(\sigma) = \sum_{i=1}^{n} w_i C_i(\sigma)$$

Q: What is $A(\sigma^*) - A(\sigma^{*\prime})$



$A(\sigma^*) = \left(\sum_i w_i C_i + w_k(T + t_k) + w_j(T + t_k + t_j)\right) + \sum_\ell w_\ell C_\ell$

$A(\sigma^{*\prime}) = \sum w_i C_i + w_j(T + t_j) + w_k(T + t_k + t_j) + \sum_\ell w_\ell C_\ell$

$A(\sigma^*) - A(\sigma^{*\prime}) = \boxed{w_j t_k - w_k t_j}$

But $\boxed{w_j/t_j > w_k/t_k}$ $\overset{\text{mult.}}{\Rightarrow}$ $\boxed{w_j t_k > w_k t_j}$ $\Downarrow$

Thus $A(\sigma^*) - A(\sigma^{*\prime}) > 0$, a contradiction because this means σ^* is not optimal. So σ is optimal.