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Subset Sum

Input: $S = \{x_1, \dots, x_k\}$, t

Output: $\{y_1, \dots, y_\ell\} \subseteq S$: $\sum_{i=1}^{\ell} y_i = t$

ex: $S =$ 1, 2, 2, 6, 7, 10

$t = 15$:

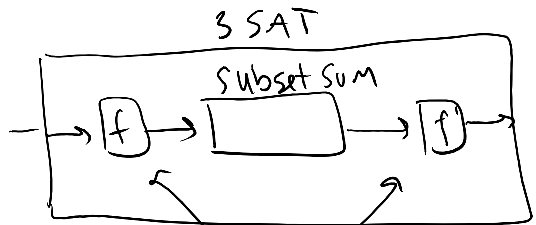
6, 7, 2

Goal: Show Subset-Sum is NP-Hard

Strategy: Prove 3-SAT reduces to subset sum

3-SAT is NP Hard, so if we show 3-SAT reduces to S-S, this means S-S is even harder than 3-SAT

1.



Create f, f'

2. Show f, f' take Polynomial time

3. S-S has soln $\iff$ 3-SAT has solution

3-SAT

Input: $X_1, \dots X_n$, variables, $c_1, \dots, c_m$ clauses each involving ≤ 3 variables

Output: assigment that satisfies all clauses

Clause: $(x_1 \vee \bar{x}_2 \vee \bar{x}_3)$

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#'s in set: $\overbrace{\qquad\qquad\qquad}^{n+m \text{ digits}}$

$C_1 = (x_1 \vee \bar{x}_2 \vee \bar{x}_3)$

		1	2	3	...	m	c_1	c_2	c_3	...	c_m
$x_1 \to w_1$	{	1	0	0	...	0	1	0		....	0
$\bar{x}_1 \to y_1$		1	0	0	...	0	0	0		....	0
$x_2 \to w_2$	{	0	1	0	...	0	0			....	0
$\bar{x}_2 \to y_2$		0	1	0	...	0	1	0		....	0
$x_3 \to w_3$	{	0	0	1	...	0	0			....	0
$\bar{x}_3 \to y_3$		0	0	1	...	0	1			....	0
				⋮				⋮			
$\tilde{C}_1$	{						1	0		......	0
							1	0		......	
$\tilde{C}_2$	{							1	0	...	0
								1	0	...	0
$\tilde{C}_3$	{								1	...	0
									1	...	0
						⋮			⋮		
t		1	1	1	...	1	3	3		....	3

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1. f: See previous

f': if w_i is in subset $x_i = 1$

if y_i is in subset $x_i = 0$

2. Polynomial time (Poly # of #'s, each with Poly digits)

3. • If 3-SAT Solution

→ Include $\begin{cases} w_i \text{ \# if } x_i = 1 \\ y_i \text{ \# if } \bar{x}_i = 0 \end{cases}$

Then every clause digit has at least 1, can add 2 more from $\tilde{c}_i$ to get to 3.

• If S-S has solution

- Note: no carry over, so if get to t,

→ one of $x_i, \bar{x}_i$ is in subset

→ without $\tilde{c}_i$'s each digit $c_1, \dots c_m$ has at least 1 contributing → each clause has at least one satisfying variable