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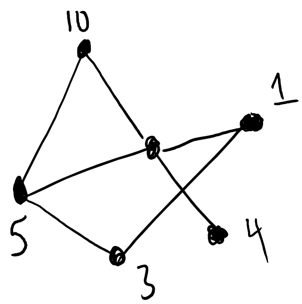
Max-Weight Independent Set Problem (MWISP)



Input: Graph (V, E) and weight function $w: V \to \mathbb{R}^+$

vertices edges

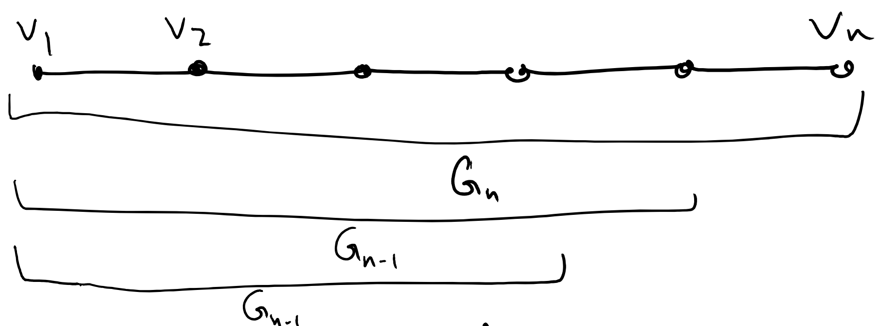
Output: • $S \subseteq V$ s.t. if $(v_i, v_j) \in E$, v_i, v_j can't both be in S. } Independent set

• S maximizes $w(S) = \sum_{v_i \in S} w(v_i)$ } max weight

This set is Max Weight Ind. Set (MWIS)

"Objective function"

MWIS



1. Think about form of Soln
2. Related soln to smaller soln

MWIS on G_n is

i) MWIS on G_{n-1}

<u>OR</u>

ii) MWIS on G_{n-2} + v_n

} Take max of these two options!

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3. Create a recurrence for objective function value:

- $A[i]$ is weight of MWIS of G_i

$$A[i] = \max\{\underbrace{A[i-1]}_{\text{case (i)}}, \underbrace{A[i-2]+w(v_i)}_{\text{(ii)}}\}$$

Q: What is $A[0]$, $A[1]$?

$A[0] = 0$, $A[1] = w(v_1)$

Fill in A using loop

$A[0] = \boxed{0}$
$A[1] = \boxed{1}$
for $i = \boxed{2 \text{ to } n:}$
  $A[i] = \boxed{\max\{A[i-1], A[i-2]+w(v_i)\}}$

Run Time: $O(n)$
Correctness: Induction

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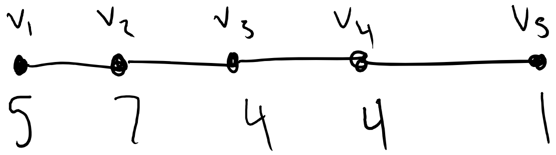
ex:



A	0	5	7	9	11	11
	0	1	2	3	4	5

$S = \emptyset$

. . . .

Case (i): $A[5] = A[4]$ * $v_5 \notin S$

Case (ii): $A[5] = A[3] + 1$

Case (i) $A[4] = A[3]$

case (ii) $A[4] = A[2] + 4$ * $v_4 \in S$

Q: Write pseudo code to get S given A

```
S = ∅
i = n
while i ≥ 0
    if  A[i] = A[i-2] + v_n
          S = S + i
          i = i - 2
    else   i = i - 1
```

} O(n) runtime!

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Indicator Random Variables Again

Interview n ppl for job, if interviewee is better than current, immediately hire.

How many hires depends on ranking of people

$\{3, 4, 1, 5, 2\}$

1st person, better → hire, worse, better → hire, worse

Suppose ranking order is random. How many hires?

Sample space Ω: all possible ranking orders (ways to order numbers $1, 2, \ldots, n$) size $= n!$

$H(s) =$ # of hires given ranking $s \in \Omega$ (random variable)

just a function!

$= \sum X_i(s)$

indicator random variables

"How can I break big random variable into a sum of random variables that take value 1 or 0?"

$$X_i(s) = \begin{cases} 1 & \text{if person } i \text{ is hired in ordering } s \\ 0 & \text{otherwise} \end{cases}$$

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$$H(s) = \sum_{i=1}^{n} X_i(s)$$

$$E[H] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i]$$

$$= \sum_{i=1}^{n} \Pr(\text{event associated with } X_i \text{ occurs})$$

$$\Pr(\text{Person } i \text{ is best so far})$$

$$\{3, 4, 1, 5, 2\}$$

1st person, better → hire, worse, better → hire, worse

Consider the first i elements. We include all permutations of these elements. Want i to be largest. $\Rightarrow$ Occurs w/prob $1/i$

$$E[H] = \sum_{i=1}^{n} \frac{1}{i} \leq \ln(n) + 1.$$