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Review

- What is a statement?
- What is a predicate?

Can combine statements to get complex statements (using logical connectors)

Let P, Q be statements:

- $P \wedge Q$: "P and Q", conjunction
  - ✲ True when both P and Q are true
- $P \vee Q$: "P or Q", "disjunction
  - ✲ True when P, Q, or both are true
- $P \rightarrow Q$: "If P, then Q", implication
  - ✲ True when P is false or both P and Q true
- $P \leftrightarrow Q$: "P if and only if Q", biconditional
  - ✲ True when both false or both true

Also $\neg P$: "Not P", negation

✲ True if P is false

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Q: Let P = "Dogs have wings", Q = "$1+1=2$"

Which are true?

1) $P \wedge Q$ 2) $P \vee Q$ 3) $P \rightarrow Q$ 4) $P \leftrightarrow Q$

5) $\neg P$

A) 2, 4, 5 B) 1, 2, 5 C) 2, 3, 5 D) 3, 5

Set Builder Notation Revisited

$$\{ \underbrace{f(x)}_{\text{function of } x} : \underbrace{H(x)}_{\text{Predicate involving } x} \}$$

= Take set of $f(x)$ where $H(x)$ is true

e.g. $\{ 2x : x \in \mathbb{N} \wedge x \text{ is odd} \}$

= Set of all numbers $2 \cdot x$ where x is an integer and x is odd

$= \{2, 6, 10, 14, 18, \ldots\}$

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Do with A

n is a multiple of two

Let $E(n)$ be the predicate "n is even"

Q: Is the following true? (Discuss)

$\boxed{\text{For all } n, \quad E(n) \rightarrow E(2n)}$

A) Yes

B) It is only true for some values of n

C) Undefined, since $E(n)$ is not a statement

D) No

If pick a specific n, get a statement

* It is true for any n

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Quantifiers : turn Predicates into Statements

Universal Quantifier : $\forall$ means "for all", "every"

ex: $\forall x, x > 0$ means "for every number x, $x>0$ is true"

Statements

Existential Quantifier: $\exists$ means "there exists" "there is"

$\exists x : x > 0$ means

"there exists a number x such that $x>0$ is true"

Q: Is the following true or false? Discuss

$$\forall x, \exists y : x = y$$

A) True

B) False

C) Not enough information to decide

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Depends on the Domains of x, y

$\underbrace{\forall x \in \mathbb{Z},}\ \underbrace{\exists y \in \mathbb{N}:}\ x = y$ False!

$\forall$ x $\in$ some set , ← end of universal quantifier

$\exists$ x $\in$ some set : ← end of existential quantifier

$x=-1$, an integer, but there is no natural number y s.t. $y=-1$.

$$\underbrace{\forall x \in \mathbb{N}, \exists y \in \mathbb{N}: x = y}_{\text{These types of statements appear often in proofs.}} \quad \text{True!}$$

ex:

Let $P(n)$ be the predicate $7^m - 1$ is divisible by 6. We will prove $\boxed{\forall n \in \mathbb{N}, \boxed{P(n)}}$ is true.

statement →

↑ predicate

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Turning English Predicates Into Math:

$P(n) =$ n cents postage can be formed from 5¢ and 8¢ stamps

$$P(n) \equiv \exists x, y \in \mathbb{Z} : x, y \geq 0 \wedge 5x + 8y = n$$

(common construction $\exists \sim : \sim \wedge \sim$)

Why? When writing proofs, usually easier to manipulate math. Useful to be able to convert from English to math

Important

- For input to Predicate NO QUANTIFIER
- For any othe variable NEED QUANTIFIER

$$S \equiv [\qquad\qquad\qquad]$$

statement

all variables quantified

$E(n) =$ n is even

$K(n) \equiv$ all integers greater than n are even

$$K(n) \equiv \forall x \in \mathbb{Z},\ x > n \rightarrow E(x)$$

(common construction: $\forall \sim ,\ \sim \rightarrow \sim$)

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Q: $m|n \equiv$ m divides n. What is $2|6$?

A) True B) False C) 3 D) $\frac{1}{3}$

$$\boxed{m|n \equiv \exists\, r \in \mathbb{Z} : m \cdot r = n}$$

PowerPoint

ex: For $n, m \in \mathbb{N}$

$R(n,m) \equiv$ every natural number less than m divides n.

$T(n,m) \equiv$ there is a natural number less than m that divides n.

$W(n,m) \equiv$ n and m don't have a common factor