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Recall Sum/Product/Subtraction

Q: There are 10 singles left in Coffrin and you and 2 friends want to pick 3 of them. How many ways could you choose rooms.

A) 30 B) 300 C) 720 D) 1000

Answer: using product rule $10 \cdot 9 \cdot 8 = 720$

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Permutations + Combinations

<u>Definition</u> K-permutation of n elements

- An ordering of a set of K elements where those K are chosen from n elements

$P: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, $P(n,k)$ = # of k-permutations of n elements

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Q:

What is a permutation?

- An ordering of a set of elements

What is a k-permutation?

- An ordering of a set of k elements

What is a formula for $P(n,k)$? Using product rule:

$$n\cdot(n-1)(n-2)\cdots(n-k+1)$$

How many permutations are there of n elements?

$$n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdots\cdot 1 = n!$$

Product Symbol / Summation Symbol

Given an ordered list of elements $(a_1, a_2, a_3, \ldots)$

$$\prod_{i=j}^{k} = a_j \times a_{j+1} \times a_{j+2} \times \cdots \times a_k$$

$$\sum_{i=j}^{k} = a_j + a_{j+1} + a_{j+2} + \cdots + a_k$$

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ex: $\prod_{i=1}^{n} i = 1 \cdot 2 \cdot 3 \cdot 4 \cdots \cdot n = n!$

$\prod_{i=n-k+1}^{n} i = (n-k+1) \cdot (n-k+2) \cdots \cdot n = P(n,k)$

Another way to write $P(n,k)$:

$$10 \cdot 9 \cdot 8 \left(\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \right) = \frac{10!}{7!}$$

so $P(n,k) = \dfrac{n!}{(n-k)!}$

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them.

Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later. How many ways could you pick 3 rooms?

A) 30 B) 120 C) 240 D) 360

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We know 720 ways if care about order.

If care about order, these are all different

$$\left\{ \begin{array}{l} (2,3,5),\ (2,5,3),\ (3,2,5),\ (3,5,2) \\ (5,2,3),\ (5,3,2) \end{array} \right.$$

My pick ↑ Friend 1 pick ↑ Friend 2 pick ↑

But if don't care about order, these are all the same. $\{2,3,5\}$

$\Rightarrow$ Over counting by a factor of 6 for each set!

$720/6 = 120$

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Function

$C(n,r) = \binom{n}{r}$ = "n choose r" is the number of sets of r elements chosen from a set of n elements.

Fact: $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

Pf: $P(n,r) = \binom{n}{r} \cdot P(r,r)$ Why?

$\Rightarrow \binom{n}{r} = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!\left(\frac{r!}{1!}\right)} = \frac{n!}{(n-r)!\,r!}$

The number of ways we can order r things chosen from among n things is equal to the number of subsets of r things, times the ways we can order each subset.

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Q: If 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

A) 40 B) 56 C) 60 D) 112

$$\frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 8\cdot 7 = 56$$