Name ____________________

CS200 - Midterm Review Questions

1. For all $x, y \in \mathbb{Z}$, prove that if $\neg 5|xy$. then $\neg 5|x$ and $\neg 5|y$. (Try proof by contrapositive combined with proof by cases.)

   Solution We prove the contrapositive. Let $x, y \in \mathbb{Z}$. Assume $5|x$ or $5|y$. We will take each of these cases separately. In the first case, if $5|x$, then there exists $m \in \mathbb{Z}$ such that $5m = x$. Thus $xy = 5my$, so $5|xy$ since my is an integer. In the second case, if $5|y$, then there exists $n \in \mathbb{Z}$ such that $5n = y$. Thus $xy = x5n$. Therefore $5|xy$, since xn is an integer.

2. Prove using a direct proof combined with proof by cases that $\forall n \in \mathbb{Z}$, $n^2 \geq n$.

   Solution We will prove that $\forall n \in \mathbb{Z}$, $n^2 \geq n$. There are three cases: $n = 0, n \geq 1$, and $n \leq -1$. In the first case, if $n = 0$, we have $0^2 = 0$, the the statement holds. In the second case, if $n \geq 1$, then multiplying both sides of the inequality by n, we have $n^2 \geq n$. In the case that $n \leq -1$, we have n^2 is positive but n is negative, so $n^2 \geq n$.

3. Let S be the set of all people. Let $G(x, y)$ be the predicate, x is the grandmother of y, for $x, y \in S$. Let $C(x, y)$ be the predicate x and y are cousins for $x, y \in S$.

   (a) Let Y be the statement: "All people have at least two grandmothers." $Y \equiv$

   (b) Let Q be the statement "Every pair of cousins share a grandmother." $Q \equiv$

   Solution

   (a) $Y \equiv \forall x \in S, \exists y, z \in S : (y \neq z) \wedge G(y, x) \wedge G(z, x)$

   (b) $Q \equiv \forall x, y \in S, C(x, y) \rightarrow (\exists z \in S : G(z, x) \wedge G(z, y))$

4. Translate into If P then Q

   (a) You will be rich only if you win the lottery.

   (b) You will be rich if you win the lottery.

   Solution

   (a) If you are rich, then you won the lottery.

   (b) If you win the lottery, then you will be rich.

5. For the statements below, give domains for which the statement is true, and for which the statement is false $\forall x, \exists y : y^2 = x$

Solution
True: $x \in \{0, 1\}$, $y \in \mathbb{Z}$
False: $x \in \mathbb{N}$, $y \in \mathbb{N}$

6. (Challenge) You meet a group of 50 orcs. You know orcs are either honest or corrupt. Suppose you know that at least one of the orcs is honest. You also know that given any two of the orcs, at least one is corrupt. Can you figure out how many of the orcs are corrupt and how many are honest? If G is the set of orcs, and $C(g)$ is the predicate, "orc g is corrupt," can you express these statements ("given any two orcs, at least one is corrupt," "at least one orc is honest") using math?