CS200 - Midterm Review Questions

1. For all $x, y \in \mathbb{Z}$, prove that if $\neg 5|xy$. then $\neg 5|x$ and $\neg 5|y$. (Try proof by contrapositive combined with proof by cases.)

2. Prove using a direct proof combined with proof by cases that $\forall n \in \mathbb{Z}$, $n^2 \geq n$.

3. Let S be the set of all people. Let $G(x, y)$ be the predicate, x is the grandmother of y, for $x, y \in S$. Let $C(x, y)$ be the predicate x and y are cousins for $x, y \in S$.

   (a) Let Y be the statement: "All people have at least two grandmothers." $Y \equiv$

   (b) Let Q be the statement "Every pair of cousins share a grandmother." $Q \equiv$

4. Translate into If P then Q

   (a) You will be rich only if you win the lottery.

   (b) You will be rich if you win the lottery.

5. For the statements below, give domains for which the statement is true, and for which the statement is false $\forall x, \exists y : y^2 = x$

6. (Challenge) You meet a group of 50 orcs. You know orcs are either honest or corrupt. Suppose you know that at least one of the orcs is honest. You also know that given any two of the orcs, at least one is corrupt. Can you figure out how many of the orcs are corrupt and how many are honest? If G is the set of orcs, and $C(g)$ is the predicate, "orc g is corrupt," can you express these statements ("given any two orcs, at least one is corrupt," "at least one orc is honest") using math?