S.KIMMEL

Today

Describe + solve problems using
- conditional probability
- random variables
- expectation value

Announcements

- Spring Symposium
- No quiz

Discuss

- Sample space
- Event
- Pr(Event)

S.KIMMEL

Conditional Probability

Let $P(E|F)$ be probability event E occured, if you know event F occured. (Conditional probability of E, given F)

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

ex: Let Sample Space $= \{0,1\}^3 = \{000, 001, 010, \ldots\}$

chosen uniformly at random

Let $F =$ first bit is 0

Let $E = 2$ consecutive zeros

What is $P(E|F)$? What is $P(E)$?

A) $\frac{3}{8}, \frac{3}{8}$ B) $\frac{1}{2}, \frac{3}{8}$ C) $\frac{2}{3}, \frac{1}{2}$ D) $\frac{1}{2}, \frac{2}{3}$

$E = \{000, 001, 100\}$ $P(E) = 3/8$

$E \cap F = \{000, 001\}$ $P(E \cap F) = 2/8$

$F = \{000, 001, 010, 011\}$ $P(F) = \frac{4}{8} \Rightarrow P(E|F) = \frac{1}{2}$

☆ If know first bit is 0, better chance of 2 consecutive 0's.

S.KIMMEL

Independent Events

$E, F \subseteq S$ are independent $\Leftrightarrow$

$$P(E) = P(E|F) = \frac{P(E \cap F)}{P(F)}$$

(Probability of E occurring is doesn't depend on whether F occured.)

Thm: If E, F are independent,

$$P(E \cap F) = P(E) \cdot P(F)$$

Pf:

$$\frac{P(E \cap F)}{P(F)} = P(E|F) = P(E)$$

ex: If flip a coin twice, what is the probability of getting 2 Heads?

A: $\Pr(H_1 \cap H_2)$ but what you get on first coin flip doesn't effect 2^{nd} coin flip.

H_1: 1^{st} outcome Heads

H_2: 2^{nd} outcome Heads

$$P(H_1 \cap H_2) = P(H_1) P(H_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

In general, need to be careful, events that seem independent might not be. <u>BUT</u> in this class, generally don't need to be careful.

S.KIMMEL

Q: Suppose you have a di where $P(6) = \frac{1}{2}$, $P(1) = P(2) = \dots = P(5) = \frac{1}{10}$. What is the probability of getting two 6's out of 4 rolls?

A. $S = \{1, 2, 3, 4, 5, 6\}^4$

$E = \{ i : i \text{ contains } 2\ 6\text{'s} \}$

$$Pr(E) = \sum_{i \in E} Pr(i)$$

Because each roll is independent:

$$Pr(6\ 6\ 1\ 2) = Pr(6) \cdot Pr(6) \cdot Pr(1) \cdot Pr(2)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{400}$$

If switch order, $Pr(i)$ is still same!

$$\Rightarrow Pr(E) = \sum_{i \in E} \frac{1}{400} = \frac{|E|}{400} = \frac{150}{400}$$

Using product rule: $|E| = \binom{4}{2} \cdot 5 \cdot 5 = 150$

places where 6 can be

choice for first non 6

choice for second non 6

S.KIMMEL

Random Variable

Function $X: S \to \mathbb{R}$

sample space

e.g. X = sum of the outcomes of 2 dice rolls

$X(1,6) = 7$

$X(2,3) = 5$

def: The expected (average) value of X is

$$\mathbb{E}[X] = \sum_{i \in S} Pr(i) X(i)$$

Q: What is the average value of one roll of our weighted di? $\left(P(6) = \frac{1}{2}, P(\text{other}) = \frac{1}{10}\right)$

A) 3

B) $\frac{6}{7}$

C) $\frac{9}{2}$

D) $\frac{27}{7}$

$$\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} + \frac{5}{10} + \frac{6}{2} = \frac{15}{10} + 3 = \frac{3}{2} + \frac{6}{2} = \boxed{\frac{9}{2}}$$