CS200 - Worksheet 2

(Taken from *Discrete Mathematics, an Open Introduction* by Levin). Read the following proofs of the statement: If ab is even, then a or b is even. However, each proof actually proves a statement of the form $P \to Q$. What is P and Q in each? What do you notice about the language? Especially consider similarities and differences regarding language, style, and structure. What words are used repeatedly, and what do those words signal to the reader?

1. Suppose a and b are odd. That is, $a = 2k+1$ and $b = 2m+1$ for some integers k and m. Then

$$\begin{aligned} ab &= (2k+1)(2m+1) \\ &= 4km + 2k + 2m + 1 \\ &= 2(2km + k + m) + 1. \end{aligned} \tag{1}$$

   Therefore, ab is odd.

2. Assume that a or b is even. Suppose it is a, since the case where b is even will be identical. That is, $a = 2k$ for some integer k. Then

$$ab = (2k)b = 2(kb). \tag{2}$$

   Therefore ab is even.

3. Suppose that ab is even but a and b are both odd. Namely, $a = 2k+1$ and $b = 2j+1$ for some integers k, and j. Then

$$\begin{aligned} ab &= (2k+1)(2j+1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1. \end{aligned} \tag{3}$$

   But this means that ab is odd, which contradicts our premise. Thus a and b can not both be odd.

4. Assume ab is even. Namely, $ab = 2n$ for some integer n. Then there are two cases: a must be either even or odd. If it is even then the statement is true. If it is odd, then $a = 2k+1$ for some integer k. Then we have

$$\begin{aligned} 2n &= (2k+1)b \\ &= 2kb + b. \end{aligned} \tag{4}$$

   Subtracting $2kb$ from both sides, we get

$$2(n - kb) = b. \tag{5}$$

   Therefore, b must be even, and the statement is true.