CS200 - Worksheet 1

A *set* is a collection of unorderd things (i.e. the order doesn't matter, and there are no repeated objects). Those things could be numbers, letters, people, minerals, other sets or any combination.

(The following is from *Discrete Mathematics, an Open Introduction* by Levin):

Set Theory Notation

$\{,\}$	We use these **braces** to enclose the elements of a set. So $\{1,2,3\}$ is the set containing 1, 2, and 3. (Roster notation)
$:$	$\{x : x > 2\}$ is the set of all x **such that** x is greater than 2. (set-builder notation)
$\in$	$2 \in \{1,2,3\}$ asserts that 2 is **an element of** the set $\{1,2,3\}$.
$\notin$	$4 \notin \{1,2,3\}$ because 4 **is not an element of** the set $\{1,2,3\}$.
$\subseteq$	$A \subseteq B$ asserts that A **is a subset of** B: every element of A is also an element of B.
$\subset$	$A \subset B$ asserts that A **is a proper subset of** B: every element of A is also an element of B, but $A \neq B$.
$\cap$	$A \cap B$ is the **intersection of** A **and** B: the set containing all elements which are elements of both A and B.
$\cup$	$A \cup B$ is the **union of** A **and** B: is the set containing all elements which are elements of A or B or both.
$\times$	$A \times B$ is the **Cartesian product of** A **and** B: the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$.
$\setminus$	$A \setminus B$ is A **set-minus** B: the set containing all elements of A which are not elements of B.
$\overline{A}$	The **complement of** A is the set of everything which is not an element of A. (Depends on what "everything" is. Define U = universal set to be everything.)
$\lvert A \rvert$	The **cardinality (or size) of** A is the number of elements in A.

The following are "famous" sets:

- $\emptyset$ = empty set = $\{\}$
- $\mathbb{N}$ = the set of natural numbers $= \{1, 2, 3, \dots\}$. (Note: in DMOI, $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$)
- $\mathbb{Z}$ = set of integers $= \{\dots, -3, -2, -1, 0, 1, 2, 3 \dots\}$
- $\mathbb{R}$ = the set of real numbers
- $\mathbb{Q}$ = the set of rational numbers

1. Let $T = \{x, y, \{g, h\}, k\}$. True or false:
   (a) $g \in T$
   (b) $\{g, h\} \in T$
   (c) $\{g, h\} \subset T$
2. Describe the following sets in roster notation (list the first few elements). If the set is also "famous" give its symbol.
   (a) $A = \{2^x : x \in \mathbb{N}\}$
   (b) $B = \{x : x \text{ is even and } x \in \{1, 3, 5\}\}$
   (c) $C = \{x \geq 0 : x \text{ is even or } x \text{ is odd}\}$
3. Write the following in set-builder notation using as concise notation as possible
   (a) $\{2, 4, 6, 8, 10, 12\}$
   (b) $\{2, 4, 8, 16, 32, 64\}$
   (c) $\{0, -1, -2, -3, \dots\}$
   (d) $\{1, 4, 9, 16, 25, 36, \dots\}$
   (e) $\{2, 4, 6, 8, 10, \dots\}$
   (f) $\{1, 3, 5, 7, 9, 11, \dots\}$
   (g) $\{1, 4, 9, 16, 25, 36, \dots\} \cap \{2, 4, 6, 8, 10, \dots\}$
   (h) $\{a, e, i, o, u\}$
4. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$
   (a) What is $A \times B$?
   (b) What is $|A \times B|$?
   (c) Is $A \subset B$?
   (d) Is $A \subseteq B$?
   (e) Is $A \subset A$?
   (f) What is $A \setminus B$?
   (g) What is $A \cup B$?
   (h) What is $A \cap B$?
5. Which of the following are the empty set:
   (a) $\{x : x \text{ is odd and } 7 < x < 9\}$
   (b) $\{0\}$
   (c) $\{\emptyset\}$
   (d) $\mathbb{Z} \cap \mathbb{Q}$
6. Let A and B be sets with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$? Explain.
7. Let $X = \emptyset$, $Y = \{\emptyset\}$, $Z = \{\{\emptyset\}\}$. Are the following true or false?
   (a) $\emptyset \in X$
   (b) $\emptyset \in Y$
   (c) $\emptyset \in Z$
   (d) $X \subseteq Y$
   (e) $Y \subseteq Z$
   (f) $X \in Y$
   (g) $Y \in Z$
8. Find sets A and B such that $A \subset B$ and $A \in B$.