CS200 - Worksheet 1

A *set* is a collection of things. Those things could be numbers, letters, people, minerals, or other sets.

(The following is from *Discrete Mathematics, an Open Introduction* by Levin):

Set Theory Notation

$\{,\}$	We use these **braces** to enclose the elements of a set. So $\{1,2,3\}$ is the set containing 1, 2, and 3. (Roster notation)
$:$	$\{x : x > 2\}$ is the set of all x **such that** x is greater than 2. (set-builder notation)
$\in$	$2 \in \{1,2,3\}$ asserts that 2 is **an element of** the set $\{1,2,3\}$.
$\notin$	$4 \notin \{1,2,3\}$ because 4 **is not an element of** the set $\{1,2,3\}$.
$\subseteq$	$A \subseteq B$ asserts that A **is a subset of** B: every element of A is also an element of B.
$\subset$	$A \subset B$ asserts that A **is a proper subset of** B: every element of A is also an element of B, but $A \neq B$.
$\cap$	$A \cap B$ is the **intersection of** A **and** B: the set containing all elements which are elements of both A and B.
$\cup$	$A \cup B$ is the **union of** A **and** B: is the set containing all elements which are elements of A or B or both.
$\times$	$A \times B$ is the **Cartesian product of** A **and** B: the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$.
$\setminus$	$A \setminus B$ is A **set-minus** B: the set containing all elements of A which are not elements of B.
$\overline{A}$	The **complement of** A is the set of everything which is not an element of A. (Depends on what "everything" is. Define U = universal set to be everything.)
$\lvert A \rvert$	The **cardinality (or size) of** A is the number of elements in A.

The following are "famous" sets:

- $\emptyset$ = empty set = $\{\}$
- $\mathbb{N}$ = the set of natural numbers = $\{1, 2, 3, \dots\}$. (Note: in DMOI, $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$)
- $\mathbb{Z}$ = set of integers = $\{\dots, -3, -2, -1, 0, 1, 2, 3 \dots\}$
- $\mathbb{R}$ = the set of real numbers
- $\mathbb{Q}$ = the set of rational numbers

1. Let $T = \{x, y, \{g, h\}, k\}$. True or false:
   (a) $g \in T$
   (b) $\{g, h\} \in T$
   (c) $\{g, h\} \subset T$

   Solution

   (a) False
   (b) True
   (c) False

2. Describe the following sets in roster notation (list the first few elements). If the set is also "famous" give its symbol.
   (a) $A = \{2^x : x \in \mathbb{N}\}$
   (b) $B = \{x : x \text{ is even and } x \in \{1, 3, 5\}\}$
   (c) $C = \{x \geq 0 : x \text{ is even or } x \text{ is odd}\}$

   Solution

   (a) $A = \{1, 2, 4, 8, 16, \dots\}$
   (b) $B = \{\} = \emptyset$
   (c) $C = \{0, 1, 2, 3, 4, \dots\} = \mathbb{N} \cup \{0\}$

3. Write the following in set-builder notation using as concise notation as possible
   (a) $\{2, 4, 6, 8, 10, 12\}$
   (b) $\{2, 4, 8, 16, 32, 64\}$
   (c) $\{0, -1, -2, -3, \dots\}$
   (d) $\{1, 4, 9, 16, 25, 36, \dots\}$
   (e) $\{1, 3, 5, 7, 9, 11, \dots\}$
   (f) $\{1, 4, 9, 16, 25, 36, \dots\} \cap \{2, 4, 6, 8, 10, \dots\}$
   (g) $\{a, e, i, o, u\}$

Solution There are many correct solutions.

(a) $\{2x : 1 \leq x \leq 6\}$
(b) $\{2^x : 1 \leq x \leq 6\}$
(c) $\{-|x| : x \in \mathbb{Z}\}$ or $\{x : x \leq 0, x \in \mathbb{Z}\}$
(d) $\{x^2 : x \in \mathbb{N}\}$ or $\{x : \sqrt{x} \in \mathbb{N}\}$
(e) $\{2x - 1 : x \in \mathbb{N}\}$
(f) $\{(2x)^2 : x \in \mathbb{N}\}$ or $\{x : x$ is an even square$\}$

4. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$

(a) What is $A \times B$?
(b) What is $|A \times B|$?
(c) Is $A \subset B$?
(d) Is $A \subseteq B$?
(e) Is $A \subset A$?
(f) What is $A \setminus B$?
(g) What is $A \cup B$?
(h) What is $A \cap B$?

Solution

(a) $A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$
(b) $|A \times B| = |A| \times |B| = 6$.
(c) Yes. Both 1 and 2 and elements of B.
(d) Yes. Both 1 and 2 and elements of B.
(e) No. $\subset$ can only be used when the two sets are not equal.
(f) $\emptyset$.
(g) B. B already contains all the elements of A, so adding those elements doesn't do anything
(h) A. The elements of A are in both. Only $3 \in B$ but $2 \notin A$.

5. Which of the following are the empty set:

(a) $\{x : x$ is odd and $7 < x < 9\}$
(b) $\{0\}$
(c) $\{\emptyset\}$
(d) $\mathbb{Z} \cap \mathbb{Q}$

Solution: Only the first

6. Let A and B be sets with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$? Explain.

Solution $7 = |A \cup B| = |A \cap B| + |A \setminus B| + |B \setminus A|$. But $|A \setminus B| = |B \setminus A|$ because $|A| = |B|$, so $|A \setminus B| = 2$ and $|A| = |A \cap B| + |A \setminus B| = 5$.

7. Let $X = \emptyset$, $Y = \{\emptyset\}$, $Z = \{\{\emptyset\}\}$. Are the following true or false?

   (a) $\emptyset \in X$

   (b) $\emptyset \in Y$

   (c) $\emptyset \in Z$

   (d) $X \subseteq Y$

   (e) $Y \subseteq Z$

   (f) $X \in Y$

   (g) $Y \in Z$

   Solution

   (a) False

   (b) True

   (c) False

   (d) True

   (e) False

   (f) True

   (g) True

8. Find sets A and B such that $A \subset B$ and $A \in B$.

   Solution For example, $A = \{1, 2\}$, $B = \{1, 2, 3, 4, \{1, 2\}, 5\}$.

9. Does the empty set contain itself?

   Solution No. The empty set contains nothing. If it contained the empty set, then it would no longer by empty!