CS200 - Problem Set 7

Due: Monday, April 9 to Canvas before class

1. Big-O Proofs

   (a) **[11 points]** Prove that $5 - 10x + 2x^2 = O(x^2)$.

   (b) **[11 points]** Prove that $\log_3(n^2) = O(\log_2(n))$. (This is a good question to review properties of logarithms. We will be using logarithms later in this class, and also they are important in many areas of computer science. Hints: recall if $\log_b(c) = x$ this means $b^x = c$. As a consequence, $a = b^{\log_b(a)}$. Also, $\log_b(a \times c) = \log_b(a) + \log_b(c)$. To prove this result, try to change the base of the term $\log_3(n^2)$ from 3 to 2. If you are feeling uncomfortable with this problem, go online and find extra practices problems dealing with logarithms and exponentiation.)

   (c) **[11 points]** Prove the following statement is false: $2^{2n} = O(2^n)$. (Hint: try a proof by contradiction!)

2. **[6 points]** Explain how you could use a graph to represent e-mail messages sent between employees at at company. What should the vertices and edges represent? Should edges be directed or not directed? Should there be self-loops in the graph (edges from one vertex back to itself)? Should there be multiple edges allowed between two vertices?

3. Consider graphs on the vertices $\{a, b, c, d, e, f\}$ such that each vertex is connected to exactly one other vertex by an edge.

   (a) **[6 points]** How many possible undirected graphs are there that satisfy the above condition? (Hint: first think about how many choices you have for the vertex connected to a.)

   (b) **[6 points]** If the graph is directed, how does your answer to part (a) change?

   (c) **[3 points]** Come up with a real world application of such a graph, where knowing the number of possible graphs you could create would be helpful.

4. Using binomial coefficients (and perhaps some other counting rules), determine how many bit strings of length 10 have

   (a) **[6 points]** exactly three 0s?

   (b) **[6 points]** at least seven 1s?

   (c) **[6 points]** exactly three zeros or start with a 1?

   (Your answer may contain terms of the form $\binom{a}{b}$)

5. How long did you spend on this homework?