# CS200 - Problem Set 2

Due: Monday, Feb. 26 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. *Don't forget induction!* **[11 points]**

   Prove that Algorithm 1 correctly searches an array of integers for a specific integer. Hint: let $P(n)$ be the predicate: `Search` works correctly on an input array of size $n$. Hint: take a look at the previous week's solution to remind your self about the general strategy for algorithms.

   **Algorithm 1:** `Search`$(s, A)$

   **Input :** Integer $s$, and an array of integers $A$

   **Output:** Returns $i$ such that $A[i] = s$, or $-1$, if $s$ is not in the array. (The first element of $A$ is at position 1.)

```
1  n = length of A;
   /* Base Case                                                    */
2  if n == 1 then
3  |  if A[1] == s then
4  |  |  return n;
5  |  else
6  |  |  return −1;
7  |  end
   |  /* Recursive case:                                          */
8  else
9  |  if A[n] == s then
10 |  |  return n;
11 |  else
12 |  |  return Search(s, A[1 : n − 1]);
13 |  end
14 end
```

2. *Set Builder to Roster Notation* **[2 point each]**

   The following sets are described in set builder notation. Describe each of them in roster notation, instead. The following symbols are used: $\mathbb{Z}$ denotes the set of integers; $\mathbb{R}$ denotes the set of real numbers; $\mathbb{N}$ denotes the set of natural numbers, i.e., $\mathbb{N} = \{1, 2, \dots\}$.

   (a) $\{r : r \in \mathbb{R} \text{ and } r = r^2\}$

   (b) $\{n : n \in \mathbb{N} \text{ and } n > n^2\}$

(c) $\{x : x$ is a letter in the word *accommodate*$\}$

(d) $\{z^2 : z \in \mathbb{Z}$ and $6 < z^3 < 160\}$.

(e) $\{S \subseteq \{2, 4, 6, 8\} : S \cap \{2, 4\} \neq \emptyset$ and $|S|$ is even$\}$

3. *Set builder notation* **[3 points each]** Write each of the following sets using set-builder notation:

   (a) $A = \{\ldots, 1/8, 1/6, 1/4, 1/2, 2, 4, 6, 8 \ldots\}$

   (b) $B = \{1, 2, 4, 8, 16, 32, \ldots\}$

   (c) $A \cap B$

   (d) Express the set of all sets of 2 integers such that the two numbers in the set are non-zero, have opposite signs, and the magnitude of one of them is the square of the magnitude of the other.

4. *Universal Set* **[2 points]** Let $U$, the universal set, be the set of even integers from 2 to 12 inclusive, and let $A = \{4, 6, 7, 9\}$, $B = \{2, 3, 4, 5, 7\}$. What is $\overline{A - B}$?

5. *Set Operations* **[2 points each]** Simplify each of the following expressions, where $A$ is an arbitrary set, $\emptyset$ is the empty set, and $U$ is the universal set. Hint: each answer to (a)-(h) is one of $A$, $U$, or $\emptyset$. Just write down the answer: no proof needed, no steps need be shown.

   (a) $A \cap U$

   (b) $A \cap \emptyset$

   (c) $A \cup U$

   (d) $A \cup \emptyset$

   (e) $A \cup A$

   (f) $A \cap A$

   (g) $A \cup \overline{A}$

   (h) $A \cap \overline{A}$

6. *Statements* **[3 points each]** For each of the following sentences, decide whether it is a statement, predicate, or neither, and explain why

   (a) Call me Ishmael.

   (b) The universe is supported on the back of a giant tortoise.

   (c) $x$ is a multiple of 7.

   (d) The next sentence is true.

   (e) The preceding sentence is false.

   (f) The set $\mathbb{Z}$ contains an infinite number of elements.

7. *Statements* **[2 point each]**

   **This problem has been postponed until next week's problem set!! If you've already done it, that is OK - but please include your solution for next week, too.**

   Simplify each of the following expressions, where $p$ denotes a statement, and $T$ and $F$ are the Boolean constants *true* and *false*. Hint: each answer is one of $p$, $T$, or $F$. No proof needed, no steps need be shown.

   (a) $T \wedge p$
   (b) $F \wedge p$
   (c) $T \vee p$
   (d) $F \vee p$
   (e) $p \vee p$
   (f) $p \wedge p$
   (g) $p \vee \neg p$

8. How long did you spend on this homework?