

CS200 - Problem Set 11

Not Due: Just for practice

1. **[6 points]** Let $A = \mathbb{N}$, and $B = \{1, 2\} \times \mathbb{N}$. Show $|A| = |B|$.
2. **[11 points]** Let S be the set of functions from \mathbb{N} to $\{0, 1\}$. Prove S is uncountably infinite.
3. **[11 points]** Graph coloring is an important problem in computer science. The goal is to assign to each vertex in a graph a color, but you can't assign the same color to two vertices if they are adjacent. For example, if you create a vertex for each country in the world, and put an edge between two vertices if they share a border, then a solution to graph coloring would tell you how to color the countries so that no two adjacent countries are the same color. The minimum possible number of colors needed to color a graph is called the *chromatic number* of the graph. Prove that the chromatic number of a tree with at least two vertices is 2.
4. **[11 points]** Consider a rooted binary tree where all leaves have distance (depth) d from the root and every non-leaf node has two children. This is called a depth- d perfect binary tree. Prove that there are $2^{d+1} - 1$ vertices in such a tree.