Counting Rules + Probability

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- Product Rule
- Sum Rule
- Subtraction Rule
- $P(n,k) = \frac{n!}{k!}$
- $C(n,k) = \frac{n!}{k!(n-k)!} = \binom{n}{k}$ = "n choose k"

How many DNA strings of length 4 i.e. strings in $\{C,T,A,G\}^4$, have exactly 2 C's or exactly 2 T's?

Subtraction Rule

2 C's + 2 T's − 2 C's & 2 T's

$\binom{4}{2} \cdot 3 \cdot 3 + \binom{4}{2} \cdot 3 \cdot 3 - \binom{4}{2}$

↑ 1st non C, ↑ 2nd non C

$= \binom{4}{2}(9+9-1) = \frac{4!}{2!2!} \cdot 17$

$= \frac{4 \cdot 3}{2} \cdot 17$

$= 2 \cdot 3 \cdot 17$

Probability (Discrete)

ex: What is the prob that outcome of a di is at

least 5?

1. Count all outcomes:

$$|\{1, 2, 3, 4, 5, 6\}| = 6$$

2. Count outcomes where value is ≥ 5

$$|\{5, 6\}| = 2$$

3. Take ration $\frac{2}{6} = \frac{1}{3}$

Terms

- Sample Space : set of all possible outcomes

  ex: $\{1, 2, 3, 4, 5, 6\}$

- Event : subset of sample space

  ex: $\{5, 6\}$

def: If all elements in a sample space S are equally likely, the prob. of an event E is

$$Pr(E) = \frac{|E|}{|S|}$$

Suppose you win some $ if you get 3 of 4 numbers matching in the lottery. What is the prob. that you get exactly 3 out of 4 correct.

5 7 0 1

x x x x

$S = \{0,1,2,3,4,5,6,7,8,9\}^4$

$E = \{x \in S : x \text{ matches } 5701 \text{ in 3 positions}\}$

$\Pr(E) = \frac{|E|}{|S|} = \frac{36}{10^4}$ $\qquad \binom{4}{3} \cdot 9 = \frac{4!}{3!1!} = 4 \cdot 9$

- Prob. that an event E does NOT happen

$$1 - \Pr(E)$$

- $E_1, E_2 \subseteq S$

$$\Pr(E_1 \cup E_2) = \frac{|E_1 \cup E_2|}{|S|} = \frac{|E_1| + |E_2| - |E_1 \cap E_2|}{|S|}$$

↑

Prob E_1 or E_2 occcurs $= \Pr(E_1) + \Pr(E_2) - \Pr(E_1 \cap E_2)$

What happens if all outcomes are <u>not</u> equally likely?

- If you have a loaded di, what is sample space?

$$\{1,2,3,4,5,6\}$$

→ Create $Pr : S \to \mathbb{R}$

- $\forall\ s \in S : 0 \le Pr(s) \le 1$
- $\sum_{s \in S} Pr(s) = 1$.

Event: $E \subseteq S$

$$Pr(E) = \sum_{s \in E} Pr(s)$$

ex: $\{1, 2, 3, 4, 5, 6\}$

$Pr(1) = Pr(2) = Pr(3) = \frac{1}{12}$

$Pr(4) = Pr(5) = Pr(6) = \frac{1}{4}$

$1 - \frac{3}{4} = \frac{9}{12}$

$E = \{5, 6\}$ $\quad Pr(E) = \sum_{s \in \{5,6\}} Pr(s) = Pr(5) + Pr(6) = \frac{1}{2}$