CS200 - Problem Set 3

1. Last week, you read section 5.3 of Proof by Richard Hammack and then found errors in the following proof of: If n is even, then n^2 is even.

   Proof:

   1. Let $n =$ an integer.
   2. Suppose n is even.
   3. Then $n = 2k$.
   4. $n^2 = (2k)^2$, $(2k)^2 = 4k^2$, so $4k^2 = 2(2k^2)$
   5. Since $(2k^2)$ is an integer, I've shown it is even.

   (The sentences in the proof are numbered to make it easier to reference specific lines in your answer.)

   This week, please rewrite the proof so that it follows Hammack's mathematical writing guidelines. (The focus here is on style, since the logic of the above proof is correct.)

2. *Party-trick Proof* Suppose you are at a party with 19 acquaintances (so there are 20 people at the party). Prove (using a proof by contradiction) that there must be at least two people at the party who talked to the same number of people over the course of the evening. (Note: we assume that if Yan talked to Jan, that also means that Jan talked to Yan. Note: my choice of 20 is not important. Your proof approach should also work for any number of party-goers.)

3. **[11 points]** The pigeonhole principle is an extremely important tool in computer science (see this StackExchange post for just some of its many diverse applications). It states: If you put at least $n + 1$ pigeons in n cubbies, there must be a cubby with more than one pigeon in it. Write two versions of the proof, one using the contrapositive, and one using contradiction. (For contradiction, remember $\neg(P \to Q) = P \wedge \neg Q$.) (Hint: both of these proofs can be very short and don't have to involve much math. In this situation I think it can be easier to explain in English.)

4. Prove $\forall n \in \mathbb{Z}$, n is even if and only if $5n + 3$ is odd. Prove one direction using a direct proof, and one direction using a contrapositive proof.

5. Suppose there is a currency called q-coins, where there are two types of coins: 3-cent coins and 7-cent coins. Suppose someone has n cents worth of q-coins, where $n \geq 14$. Prove that that person must have at least three 3-cent coins, or at least two 7-cent coins. (Use a proof by cases. For hint, see last page.)

6. How long did you spend on this homework?

Hint for problem 5: We know the number of 7-cent coins must be an integer greater than or equal to 0. Consider two possible cases: when the number of 7 cent coins is less than some amount, and when the number of 7 cent coins is more than some amount.