

# Strong Induction

Prove: It takes  $n-1$  breaks to reduce an  $n$ -square chocolate bar to  $n$  individual squares

## Set-up and Base case:

Let  $P(n)$  be the predicate: it takes  $n-1$  breaks to reduce an  $n$ -square chocolate bar to  $n$  individual pieces. We will prove  $P(n)$  is true for all  $n > 0$  using strong induction.

Base case: When you have a 1-square chocolate bar, it requires 0 breaks because it is already in 1 individual piece. Thus  $P(1)$  is true

## Inductive Step

Inductive step: We assume for induction the  $P(k)$  is true for  $1 \leq k < n$ . We will prove  $P(n)$  is true. Since  $n > 1$ , we can break our chocolate into two pieces, one with  $a$  squares, and one with  $b$  squares, where  $a + b = n$ , and  $1 \leq a < n$  and  $1 \leq b < n$ . Using our inductive assumption, it requires  $a - 1$  breaks to separate the first piece, and  $b - 1$  breaks to separate the second piece. Adding up all the breaks, we have  $(a - 1) + (b - 1) + 1 = n - 1$  breaks.