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Functions

- Big-Ω, Big-Θ
- Turning English into math

Problem with Big-O:

- $x + 1 = O(x^2)$ $\Leftarrow$ correct because big-O is <u>upper</u> bound.

- Need asymptotic <u>lower</u> bound! (Big-O is asymptotic upper bound)

"big-Omega"

<u>def</u>: Let $f, g: \mathbb{N} \to \mathbb{R}$ Then $f(x)$ is $\Omega(g(x))$ if there exist constants $k \in \mathbb{Z}$ and $C \in \mathbb{R}$ such that when $x \geq k$, then

$$f(x) \geq C \cdot g(x)$$

ex: $5x + 6 = \Omega(x)$

<u>Pf</u>: Note $\forall x \in \mathbb{N}, (x > 1) \to (6 > -x)$. Thus

$$5x + 6 \geq 5x - x \geq 4x$$

So with $k = 1$, $C = 4$, we have $5x + 6 = \Omega(x)$

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Big-O = "asymptotic upper bound"
Big-Ω = "asymptotic lower bound"

For linear search, what is a (1) best-case asymptotic lower bound (2) worst-case asymptotic lower bound?

A) $\Omega(1)$, $O(1)$ B) $\Omega(n)$, $O(1)$ C) $\Omega(1)$, $O(n)$

D($\Omega(n)$, $O(n)$).

In the best-case, the item we are searching for is at beginning of list, so we are done in constant # of steps. Worst case, need to go through the whole list, takes time $O(n)$.

* Asymptotic $\neq$ worst-case

Worst-case asymptotic
Best-case asymptotic
Average-case asymptotic

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Problem with big-Ω:

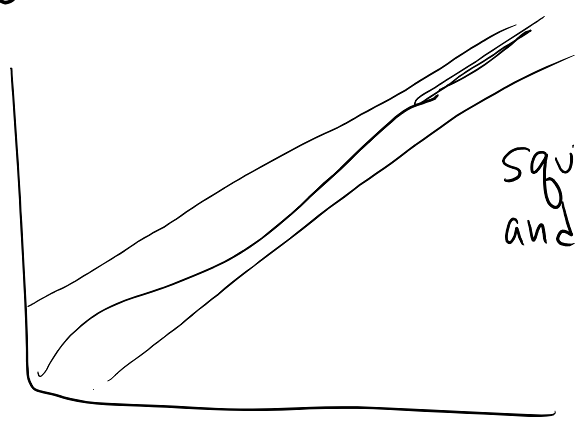
$$x^2+1 = \Omega(x)$$

"big Theta"

def: $f(x) = \Theta(g(x)) \iff f(x) = O(g(x)) \wedge f(x) = \Omega(g(x))$

ex: $x^2+1 = \Theta(x^2)$ but $x^2+1 \neq \Theta(x)$, $x^2+1 \neq \Theta(x^3)$

Big-Θ: <u>tight</u> asymptotic bound



squished between upper and lower bounds

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Predicates, Quantifiers, & English → Math.

$P(n) \equiv$ n is prime

Predicate is a function! $P: \mathbb{R} \to \{\text{true}, \text{false}\}$

$\mathbb{Z}$

$\mathbb{N}$

Not clear what domain is

For a natural number n, $P(n) \equiv$ n is prime

or $P(n) \equiv$ the natural number n is prime.

Now clear: $P: \mathbb{N} \to \{\text{true}, \text{false}\}$

Important

- For input to Predicate NO QUANTIFIER
- For any other variable NEED QUANTIFIER

e.g. $P(n) \equiv \forall n : (n \text{ is prime})$ ✗

$P(5) \equiv \forall 5 : (5 \text{ is prime})$ ✗

$P(n) \equiv \neg \exists m \in \mathbb{N} : 1 < m < n \wedge m | n$

Need to quantify m

No quantifier for n.

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$S \equiv [\qquad\qquad]$ every variable here should be quantified.

no input variable

$\forall x, \underline{\ x > 5 \rightarrow x^2 = 10\ }$

$\exists x : \underline{\ x = 2\ }$

Predicate involving x } True or false depending on value of x. Should be true to make sense

- For all x such that $x > 5$, $T(x)$

Before: $\forall x : x > 5, T(x)$

Better: $\forall x, x > 5 \rightarrow T(x)$

- $m|n$ ← true or false, not $m|n = 2$

"m divides n"

$m|n \equiv \exists r \in \mathbb{Z} : m \cdot r = n$ (Domain n, m is integers)

- $\forall x_1, x_2 \in \mathbb{R}, \quad \ldots \quad \wedge x_1 \neq x_2$ is always FALSE, because not all 2 real #'s equal each other

Better: $\forall x_1, x_2 \in \mathbb{R}, x_1 \neq x_2 \rightarrow \ldots$

We don't have this problem for $\exists$ → $\exists x_1, x_2 \in \mathbb{R} : \quad \ldots \quad \wedge x_1 \neq x_2$ OK!

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ex: For $n, m \in \mathbb{N}$

$R(n,m) \equiv$ every natural number less than m divides n.

$T(n,m) \equiv$ there is a natural number less than m that divides n.

$W(n,m) \equiv$ n and m don't have a common factor

- $R(n,m) \equiv \forall p \in \mathbb{N},\ p < m \rightarrow p \mid n$
- $T(n,m) \equiv \exists p \in \mathbb{N} : p < m \wedge p \mid n$
- $W(n,m) \equiv \neg \exists p \in \mathbb{Z} : p \mid n \wedge p \mid m$

Rewrite
- $\neg \exists x : P(x)$ using $\forall$
- $\neg \forall x, P(x)$ using $\exists$

- $\neg \exists x : P(x) \equiv \forall x, \neg P(x)$
- $\neg \forall x, P(x) \equiv \exists x : \neg P(x)$